Book of Abstracts

FUNCTIONAL ANALYSIS AND ITS APPLICATIONS

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On a generalization of difference operator over the sequence space c_0

A. M. Akhmedov

Recently it is introduced the new generalized difference operator Δ_ν [1]:

$$\Delta_\nu = \begin{bmatrix} \nu_0 & 0 & 0 & \cdots \\ -\nu_0 & \nu_1 & 0 & \cdots \\ 0 & -\nu_1 & \nu_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

where $\nu = (\nu_k)$ is either constant or strictly decreasing sequence of positive real numbers such that

$$\lim_{k\to\infty} \nu_k = L > 0 \text{ and } \sup_k \nu_k \leq 2L.$$

In [1] the fine spectrum of the operator Δ_ν on c_0 (null space of sequences) has been examined. In present work we have shown that the results of [1] concerning to the spectrum of Δ_ν are simple corollaries of the corresponding results of the work Theorem 1.5 [2] in case when (ν_k) is a constant sequence.

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Modulus of the continuity of the harmonic mappings on the boundary of the unit sphere and on the unit ball

Miloš Arsenović

We prove the sufficient conditions on the modulus of continuity of a continuous function, such that its harmonic continuation (up to a multiplicative constant) has the same module of the continuity.

Some topological and geometric properties of the domain of the generalized difference matrix $B(r,s)$ in the sequence space $\ell(p)$

Cafer Aydin and Feyzi Başar

The sequence space $\ell(p)$ was introduced by Maddox [Spaces of strongly summable sequences, Quart. J. Math. Oxford (2)**18**(1967), 345–355]. In the present study, the sequence space $\widehat{\ell}(p)$ of non-absolute type has been studied which is the domain of the generalized difference matrix $B(r,s)$ in the sequence space $\ell(p)$. Furthermore, the α-, β- and γ-duals of the space $\widehat{\ell}(p)$ have been determined, and the Schauder basis has been given. The classes of matrix transformations from the space $\widehat{\ell}(p)$ to the spaces ℓ_∞, c and c_0 have been characterized. Additionally, the characterizations of some other matrix transformations from the space $\widehat{\ell}(p)$ to the Euler, Riesz, difference, etc., sequence spaces have been obtained by means of a given lemma. The last section of the work have been devoted to some results about the rotundity of the space $\widehat{\ell}(p)$.

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On the Riesz B-difference sequence spaces

Metin Basarir

In the present paper, we define the Riesz B-difference sequence space $r^q(p, B(r, s))$ which is generalized the sequence space $r^q(p)$, the Riesz sequence space given by Altay and Basar and $r \neq 0$, $s \neq 0$. We give some topological properties and compute the α-,β- duals of this spaces.

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For the nulles of the solutions of the elementary Vekua equations

Slagjana Brsakoska, Borko Ilievski, Dragan Dimitrovski

After the year of 2000 it is noticed a huge trend in the intensity of the papers which are dealing with valuation of the nulls of the complex differential equations, especially the equation of the "complex oscillations" . For the Vekua type equations, which are similar to them, in the space of two complex variables we have not noticed such trend in the study of the nulls of the solutions. In this paper we are giving theorems for existence of the nulls of the elementary Vekua equation: and we are proving that this problem is not trivial or easy.

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A study on statistical convergence

H. Cakalli

Abstract. A characterization of statistical convergence of sequences in topological groups is obtained, and extensions of a decomposition theorem, a completeness theorem and a Tauberian theorem to the topological group setting are proved.

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On the norm and circumferences of astroids

Mehmet Can

To the time various approximations are provided to approximate the circumference of ellipses. One of them relies on the distance induced by the metric of the norm. In this article Necat Tadelen (1959) estimation

$$L = (a^p + b^p)^{1/p}, \; p \approx \frac{Log(2)}{Log(\pi/2)}$$

is extended into a Taylor series which converges to the exact values of the circumferences of ellipses. Then the series is generalized and extended to astroids.

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Some conditions for convergence and subsequential convergence in terms of regularly generated sequences

Ibrahim Canak and Aumit Totur

Let (u_n) be a sequence which is a regularly generated by a sequence (α_n), where either (α_n) or $(\Delta\alpha_n) = (\alpha_n - \alpha_{n-1})$ is moderately oscillating. In this work we have investigated some conditions under which sequence (u_n) converges or converges subsequentially.

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Positive definite solution of Lyapunov equation

Aleksandar S. Cvetković

In this paper we continue investigation of the positive definite solutions of generalized Lyapunov equation initiated by R. Bhatia at all in [**?**], [**?**], Kwong [**?**] and Cvetković, Milovanović [**?**]. We switch from the finite dimensional spaces to infinite dimensional one and prove that there exists unique positive semi-definite solution of generalized Lyapunov equation. We prove that if solution exists, its positive semi-definitness can be stated, provided, certain rational function is positive semi-definite.

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The different types of solutions of some equations

Dragana S. Cvetković-Ilić

We will consider reflexive and Re-nnd solutions of the matrix equation $AXB = C$. Also, we present new representations for the general positive and real-positive solutions of the equation $axa^* = c$ in a C^*-algebra using the characterization of positivity based on a matrix representation of an element and the generalized Schur complement.

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Some fixed point results on cone metric space

Lj. Gajić, D. Ilić, V. Rakočević

We give some results for quasi-contraction on a cone metric space. Also, we present a generalization of some results of Sehgal and Guseman and Ćirić s theorem for mappings with a generalized contractive iterate at a point to cone metric spaces, in which the cone does not need to be normal.

Subdivision in polynomial spaces

Sonja Gegovska-Zajkova, Vesna Andova1 and Ljubia M. Kocić

For the Lagrange interpolation operator, a multi-subdivision scheme is established. The existence of the corresponding functional equation of Read-Bajraktarevi type is proved and used in construction of this scheme. Associated algorithms are developed and illustrated through adequate examples.

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Some identities concerning the reverse order law for the Moore-Penrose inverse

Nebojša Dinčić

we present some identities related to the reverse order law for the Moore-Penrose inverse of operators on Hilbet spaces, on the trail of the results from (Y. Tian and S. Cheng, *Some identities for Moore-Penrose inverses of matrix products*, Linear and Multilin.r Algebra (2004)).

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Compact operators on some sequence spaces related to strong Cesàro summability and boundedness

Ivana Djolović

Many sequence spaces arise from different concepts of summability. Recent results obtained by Altay, Başar and Malkowsky [Matrix Transformations on Some Sequence Spaces Related to Strong

Cesaro Summability and Boundedness , *Appl.Math.Comput.*, doi:10.1016/j.amc.2009.01.062] are related to strong Cesàro summability and boundedness. They determined $\beta-$duals of the new sequence spaces and characterized some classes of matrix transformations on them. Here, we will present new results supplementing their research with the characterization of classes of compact operators on those spaces.

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On the generalization of the Moore-Penrose inverse

Dragan S. Djordjević

We consider the generalized Moore-Penrose inverse of a Hilbert space operator using generalized projections, instead of the orthogonal projections. Some new results are proved.

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Simple invariant subspaces of linear operator

Slaviša V. Djordjević

Let X be Banach spaces, then $\mathcal{B}(X)$ denotes the space of all bounded linear operators from X to X. Let $\pi_0(T)$ denote the set of Riesz points of T (i.e., the set of isolated eigenvalues of T of finite algebraic multiplicity) and let $\pi_{00}(T)$ denote the set of eigenvalues of T of finite geometric multiplicity (i.e. $0 < n(T-\lambda) < \infty$). $\lambda \in \pi_0(T)$ such that its algebraic multiplicity is 1 is called a simple eigenvalue (pole) of T. Let $P_1(X)$ denote the collection of subspaces of X of dimension 1. The *manifold of proper elements of* X is the space

$$Eig(X) = \{(\lambda, L, A) \in \mathbf{C} \times P_1(X) \times \mathcal{B}(X) : A(L) \subset L \text{ and } A_{|L} = \lambda I\}.$$

In the another words, the proper elements of X is a triple of an eigenvalues, invariant subspace of an operator A generalized with one eigenvector of λ and an operator A. For $(\lambda_0, L_0, A_0) \in Eig(X)$, where $L_0 = \mathcal{L}(\{x_0\})$, the operator A_0 induces an operator $\widehat{A_0}$ from quotient X/L_0 to itself, i.e. $\widehat{A_0}(x + L_0) = A_0(x) + L_0$.

In talk we will show that λ_0 is a simple pole of A_0 if and only if $\lambda_0 \notin \sigma(\widehat{A_0})$. Follows this concept we can define simple invariant subspaces of linear operator T like invariant subspace E such that $\sigma(T_E) \cap \sigma(\widehat{T_E}) = \emptyset$, where $T_E : E \to E$ the restriction of T on E, and by $\widehat{T_E}$ the operator $\widehat{T_E}(\pi(y)) = \pi(T(y))$ on the quotient space X/E and π is the natural homoeomorphism between X and X/E.

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Subspaces with a common complement in a Banach space

Dimosthenis Drivaliaris

We will discuss the problem of the existence of a common algebraic complement for closed subspaces of a Banach space. If $\mathcal{M}$ and $\mathcal{N}$ are closed subspaces of a Banach space X, then we say that $\mathcal{M}$ and $\mathcal{N}$ have a common algebraic complement if there exists a closed subspace $\mathcal{K}$ of X such that:

$$\mathcal{M} \oplus \mathcal{K} = X = \mathcal{N} \oplus \mathcal{K}.$$

We will present characterizations of such pairs of subspaces, which we proved in a recent paper (D. Drivaliaris, N. Yannakakis, Subspaces with a common complement in a Banach space, Studia Mathematica 182 (2007) 141-164). We will also give some possible applications to problems related to generalized inverses of operators.

Remark: Joint work with Nikos Yannakakis.

The Browder and Weyl spectra of an operator and its diagonal

B. P. Duggal, S. V. Djordjević and M. Chō

If $T \in B(\mathcal{X})$ is a Banach space operator and E is a closed T-invariant subspace of $\mathcal{X}$, then the restriction map $A = T|_E$ and the quotient map $B = T|_{\mathcal{X}/\mathcal{E}}$ are well defined operator in $B(E)$ and $B(\mathcal{X}/\mathcal{E})$, respectively. It is proved that: (i) If $\sigma_x(T) = \sigma_x(A) \cup \sigma_x(B)$, where σ_x is either the Weyl spectrum σ_w or the Weyl essential approximate point spectrum σ_{aw}, then $\sigma(T) = \sigma(A) \cup \sigma(B)$; (ii) if $\sigma_{aw}(T) = \sigma_{aw}(A) \cup \sigma_{aw}(B)$, and A^* has SVEP (the single–valued extension property), then $\sigma_a(T) = \sigma_a(A) \cup \sigma_a(B)$; (iii) if $\sigma(T) = \sigma(A) \cup \sigma(B)$, then a point λ is a pole (resp., finite rank pole) of the resolvent of T if and only if λ is a pole (resp., finite rank pole) of the resolvents of A and B. Letting σ_b and σ_{ab} denote, respectively, the Browder spectrum and the Browder essential approximate point spectrum, an operator $S \in B(\mathcal{X})$ satisfies Browder's theorem (resp., a-Browder's theorem) if $\sigma_w(S) = \sigma_b(S)$ (resp., $\sigma_{aw}(S) = \sigma_{ab}(S)$); S satisfies Weyl's theorem if $\sigma(S) \setminus \sigma_w(S) = \{\lambda \in \operatorname{iso}\sigma(S) : 0 < \dim(S-\lambda)^{-1}(0) < \infty\}$. Recall that S is isoloid if $\lambda \in \operatorname{iso}\sigma(S)$ implies $0 < \dim(S-\lambda)^{-1}(0)$. We prove that: (iv) if $\sigma_w(T) = \sigma_w(A) \cup \sigma_w(B)$ (resp., $\sigma_{aw}(T) = \sigma_{aw}(A) \cup \sigma_{aw}(B)$), then Browder's theorem (resp., a-Browder's theorem) transfers from A and B to T; (v) if $\sigma_w(T) = \sigma_w(A) \cup \sigma_w(B)$, and A, B are isoloid, then Weyl's theorem transfers from A and B to T.

Semi-Fredholm properties of operators in (complex) interpolation spaces

K.-H. Förster and K. Günther

As a main result we show that for interpolation morphisms S and the complex interpolation method the set of all $\theta \in]0,1[$ such that $S_{[\theta]}$ is a semi-Fredholm operator is open and the nullities, deficiencies and indices of $S_{[\theta]}$ are locally constant. This generalizes results of E. Albrecht(1984); for the proof we use ideas of E.Albrecht and V. Müller(2000).
Further we discuss semi-Fredholm properties for other interpolation methods (e.g. real methods, orbit methods) and other methods of their proofs (e.g. lifted graphs).

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1973 and all that

Robin Harte

A small surprise in spectral theory.

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Polynomial approximation of the Moore-Penrose inverse of an operator

Sotirios Karanasios

Let T be an operator with closed range from a Hilbert space into itself. We will discuss the problem of the approximation of the Moore-Penrose inverse $T^\dagger$ of T by polynomials in T. We will give conditions (necessary and necessary and sufficient) for $T^\dagger$ to belong in the weak closure and

in the norm closure of the algebra $\mathcal{A}(T, I)$ generated by T and the identity operator. Our work generalizes existing results by Antonevich, Appell, Prokhorov and Zabrejko [1], Bravo [2], Erdos [3], [4] and Feintuch [5], [6], [7], [8] on the polynomial approximation of the inverse of an invertible operator.

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Remark: Joint work with Dimosthenis Drivaliaris and Dimitrios Pappas.

Introducing of affine invariance to IFS

Ljubia M. Kocić, Elena Babač and Sonja Gegovska-Zajkova

The original definition of the IFS with affine contractive mappings is an important and handy tool for constructive approach to fractal sets. But, in spite of clarity in definition, the concept of AIFS does not allow many possibilities in the sense of modelling of such sets, typically being fairly complicated. One step in direction of improving the concept of IFS consists in introducing AIFS, a variant of IFS that permits affine invariance property which is vital from the point of modeling. The theory is supported by comprehensive examples.

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Bilipsitz mappings between sectors in planes and quasi-conformity

Vesna Manojlović

We investigate properties of the bilipsitzy with respect to the j metric and quasi-hyperbolic metric. We also investigate behavior of the bilipsitz constant in the case when the constant of the quasi-conformity tends to 1.

Characterization of EP, normal and Hermitian elements in rings

Dijana Mosić and Dragan S. Djordjević

We present characterizations of EP elements in rings with involution in purely algebraic terms and considerably simplify proofs of already existing characterizations. We also give several char-

acterizations of Moore-Penrose-invertible normal and Hermitian elements in rings with involution and the proofs are based on ring theory only.

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On the reflexivity deficit

Vladimir Müller

Let X, Y be complex vector spaces and let $\mathcal{A} \subset \mathcal{L}(X, Y)$ be a subspace. The reflexive hull of $\mathcal{A}$ is the set of all operators $T : X \to Y$ satisfying $Tx \in \mathcal{A}x := \{Ax : A \in \mathcal{A}\}$ for all $x \in X$.

We show that if $\dim \mathcal{A} = n < \infty$ then the reflexive hull of $\mathcal{A}$ is at most $\frac{n(n+1)}{2}$ dimensional; the estimate is optimal.

The main tool is the following result which is a generalization of the non-emptiness of the spectrum of a square matrix (for $k = 2$ and $m = n$): if $\mathcal{A} \subset M_{m\times n}$ is a k-dimensional subspace with $k \leq n + 1$ then

$$\dim \bigcap_{A\in\mathcal{A}, A\neq 0} \operatorname{Im} A \leq n - k + 1.$$

(joint work with C. Ambrozie and B. Kuzma)

Inner generalized inverses with prescribed idempotents in rings and their matrix representation

Biljana Načevska and Dragan S. Djordjević

We present some additive results on generalized inverses in rings. Particulary we define an inner generalized inverse with prescribed idempotents. These classes of generalized inverses are natural algebraic extension of generalized inverses of linear operators with prescribed range and kernel and we get some of their properties as an application. Also, we propose a matrix representation of an inner invertible elements in rings.

On the special transforms and Hankel determinants of several number sequences

Marko D. Petković, Predrag M. Rajković

A lot of papers about importance of Hankel determinants and various techniques for their evaluation were written. Here, we deal with Hankel determinants mostly arising in combinatorial analysis and discuss methods which lead to closed-form expression of Hankel determinants for some special number sequences. We consider the special number sequence whose Hankel determinant computation requires the usage of discrete Sobolov orthogonality. We also deal with the other special transformations of number sequences (Catalan, Binomial, Ballot, etc). A number of other interesting results and examples are given.

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Wave front sets through Gabor expanions

Stevan Pilipović

First, we introduce a descrite version of the wave fron set for Fourier-Lebesgues spaces and show the equivalence with the classical notion of the wave front set. Specifying a Gabor wavelet, we are able to describe Fourier-Lebesgue wave front of a sygnal developed into a corresponding series.

Remark: Joint results of S. Pilipović, J.Toft and N. Teofanov.

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A comparison between the concepts of quasi and almost orthogonal polynomials

Predrag M. Rajković and Sladjana D. Marinković

The generation of orthogonal functions in technical sciences (for example, in signal approximation and design of the electronic systems which generate the orthogonal signals), sometimes leads to the polynomials which are not quite orthogonal, but rather *almost orthogonal*. From those reasons, in our previous papers, we have defined a few classes of almost orthogonal polynomials. The question of its comparison with quasi-orthogonal concept arises naturally and we will pay our attention on it through this paper. It will be illustrated by numerous examples.

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On the spectral radius of linear combinations of two projections in C^*-algebras

Vladimir Rakočević and Julio Benítez

In this work we study the spectrum and give estimations for the spectral radius of linear combinations of two projections in C^*-algebra. We also study the commutator of two projections.

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Some fixed point theorems on cone metric and modular spaces

Abdolrahman Razani

In order to replace the real numbers by ordering Banach space, cone metric spaces are defined. On the other hand, in connection to ordered space theory, the theory of modular spaces is introduced. In this talk, some fixed point theorems on cone metric and modular spaces are presented. These theorems are new generalizations of famous contraction condition $d(T(x), T(y)) \leq kd(x, y)$.

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Some sequence spaces with an index defined by a modulus function

Ekrem Savas

In this paper we define the following sequence spaces by using a modulus function

$$V_p(f) = \left\{ x : \sum_m m^{p-1} f(|t_{mn}|^p) \text{ converges uniformly in } n \right\},$$

$$W_p(f) = \left\{ x : \sup_n \sum_m m^{p-1} f(|t_{mn}|^p) < \infty \right\},$$

where $p \geq 1$ and

$$t_{mn} = \frac{1}{m(m+1)} \sum_{\nu=1}^{m} \nu(x_{n+\nu} - x_{n+\nu-1}).$$

We also get some inclusion relations. Note that if $f(x) = x$, then we get $V_p(f) = V_p$ and $W_p(f) = W_p$. If $p = 1$ then $V_p = V$, reduces to the space of all sequences of almost bounded variation.

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Summability factor theorem for generalized absolute Cesàro summability

Hamdullah Şevli

Let σ_n^{α} denote the nth terms of the transform of a Cesáro matrix (C, α) of a sequence $\{s_n\}$. In 1957 Flett [**?**] made the following definition. A series $\sum a_n$, with partial sums s_n, is said to be absolutely (C, α) summable of order $k \geq 1$, written $\sum a_n$ is summable $|C, \alpha|_k$, if

$$\sum_{n=1}^{\infty} n^{k-1} \left| \sigma_{n-1}^{\alpha} - \sigma_n^{\alpha} \right|^k < \infty. \tag{1}$$

Absolute Abel summability, written as $|A|$,was defined by Whittaker [**?**] as follows:

A series $\sum a_n$ is said to be summable $|A|$ if the series $\sum a_n x^n$ is convergent for $0 \leq x < 1$ and its sum-function $\phi(x)$ satisfies the condition

$$\int_0^1 |\phi'(x)| \, dx < \infty. \tag{2}$$

In that same paper Flett extended this result to index k by replacing condition (2) by the condition

$$\int_0^1 (1-x)^{k-1} |\phi'(x)|^k \, dx < \infty. \tag{3}$$

Thus the series $\sum a_n$ is said to be summable $|A|_k$, $k \geq 1$, if the series $\sum a_n x^n$ is convergent for $0 \leq x < 1$ and its sum-function $\phi(x)$ satisfies condition (3). He then showed that summability $|A|_k$ is a weaker property than summability $|C, \alpha|_k$ for any $\alpha > -1$.

In this paper we obtain necessary and sufficient conditions for the series $\sum a_n \lambda_n$ to be summable $|A|_k$ whenever $\sum a_n$ is absolutely summable of order k by a generalized Cesàro matrix.

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The reverse order laws for some $\{i,j,k\}$-inverses

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The necessary and sufficient conditions in terms of the ranks of the known matrices for the inclusion

$$A_n\{i,j,k\}A_{n-1}\{i,j,k\}\cdots A_1\{i,j,k\} \subset (A_1A_2\cdots A_n)\{i,j,k\}$$

are presented by means of the maximum and minimum ranks of the generalized Schur complements.

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