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Approximating common fixed points for asymptotically quasi-nonexpansive mappings in the intermediate sense in convex metric spaces

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Abstract

In this paper, we give necessary and sufficient condition for strong convergence of implicit iteration process with errors for approximating common fixed point for a finite family of asymptotically quasi-nonexpansive mappings in the intermediate sense in convex metric spaces. The results presented in this paper extend and improve some known results given in the literature (see, e.g., [10, 16, 17, 20, 21, 22]).

1 Introduction and Preliminaries

Throughout this paper, we assume that E is a metric space, $F(T_i) = \{x \in E : T_i x = x\}$ be the set of all fixed points of the mappings T_i $(i = 1, 2, \dots, N)$, $D(T)$ be the domain of T and $\mathbb{N}$ is the set of all positive integers. The set of common fixed points of T_i $(i = 1, 2, \dots, N)$ denoted by F, that is, $F = \cap_{i=1}^{N} F(T_i)$.

Definition 1.1 ([2]): Let $T : D(T) \subset E \to E$ be a mapping.

(1) The mapping T is said to be L-Lipschitzian if there exists a constant $L > 0$ such that

$$d(Tx, Ty) \quad \leq \quad Ld(x, y), \qquad \forall x, y \in D(T). \tag{1.1}$$

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(2) The mapping T is said to be nonexpansive if

$$d(Tx, Ty) \quad \leq \quad d(x, y), \qquad \forall x, y \in D(T). \tag{1.2}$$

(3) The mapping T is said to be quasi-nonexpansive if $F(T) \neq \emptyset$ and

$$d(Tx, p) \quad \leq \quad d(x, p), \qquad \forall x \in D(T),\ \forall p \in F(T). \tag{1.3}$$

(4) The mapping T is said to be asymptotically nonexpansive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$d(T^n x, T^n y) \quad \leq \quad k_n d(x, y), \qquad \forall x, y \in D(T),\ \forall n \in \mathbb{N}. \tag{1.4}$$

(5) The mapping T is said to be asymptotically quasi-nonexpansive if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$d(T^n x, p) \quad \leq \quad k_n d(x, p), \qquad \forall x \in D(T),\ \forall p \in F(T),\ \forall n \in \mathbb{N}. \tag{1.5}$$

(6) T is said to be asymptotically nonexpansive type, if

$$\limsup_{n\to\infty} \left\{ \sup_{x,y \in D(T)} \Big(d(T^n x, T^n y) - d(x, y) \Big) \right\} \quad \leq \quad 0. \tag{1.6}$$

(7) T is said to be asymptotically quasi-nonexpansive type, if $F(T) \neq \emptyset$ and

$$\limsup_{n\to\infty} \left\{ \sup_{x \in D(T),\ p \in F(T)} \Big(d(T^n x, p) - d(x, p) \Big) \right\} \quad \leq \quad 0. \tag{1.7}$$

Remark 1.1: It is easy to see that if $F(T)$ is nonempty, then nonexpansive mapping, quasi-nonexpansive mapping, asymptotically nonexpansive mapping, asymptotically quasi-nonexpansive mapping and asymptotically nonexpansive type mapping all are the special cases of asymptotically quasi-nonexpansive type mappings.

Now, we define asymptotically quasi-nonexpansive in the intermediate sense mapping in convex metric space.

T is said be asymptotically quasi-nonexpansive in the intermediate sense mapping provided that T is uniformly continuous and

$$\limsup_{n\to\infty}\left\{\sup_{x\in D(T),\ p\in F(T)}\Big(d(T^n x,p)-d(x,p)\Big)\right\} \quad\leq\quad 0. \tag{1.8}$$

In 2001, Xu and Ori [21] have introduced the following implicit iteration process for common fixed points of a finite family of nonexpansive mappings $\{T_i\}_{i=1}^N$ in Hilbert spaces:

$$x_n \quad=\quad t_n x_{n-1}+(1-t_n)T_n x_n, \quad n\geq 1 \tag{1.9}$$

where $T_n = T_{n(modN)}$. (Here the mod N function takes values in the set $\{1,2,\dots,N\}$). And they proved the weak convergence of the process (1.9).

In 2003, Sun [17] modified the implicit iteration process of Xu and Ori [21] and applied the modified averaging iteration process for the approximation of fixed points of asymptotically quasi-nonexpansive mappings. Sun introduced the following implicit iteration process for common fixed points of a finite family of asymptotically quasi-nonexpansive mappings $\{T_i\}_{i=1}^N$ in Banach spaces:

$$x_n \quad=\quad \alpha_n x_{n-1}+(1-\alpha_n)T_i^k x_n, \quad n\geq 1 \tag{1.10}$$

where $n=(k-1)N+i, \quad i\in\{1,2,\dots,N\}$.

Sun [17] proved the strong convergence of the process (1.10) to a common fixed point in real uniformly convex Banach spaces, requiring only one member T in the family $\{T_i : i=1,2,\dots,N\}$ to be semi-compact. The result of Sun [17] generalized and extended the corresponding main results of Wittmann [20] and Xu and Ori [21].

Very recently, Saluja and Nashine [16] study the following implicit iteration process for common fixed points of a finite family of asymptotically quasi-nonexpansive mappings $\{T_i\}_{i=1}^N$ in the setting of convex metric spaces:

$$x_n \quad=\quad W(x_{n-1}, T^n_{n(mod\ N)}x_n, u_n; \alpha_n, \beta_n, \gamma_n), \quad n\geq 1 \tag{1.11}$$

where $\{u_n\}$ is a bounded sequence and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are three sequences in $[0,1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ for $n = 1, 2, \ldots$. They gave the necessary and sufficient condition for strong convergence of said iteration scheme and mappings. The result of [16] extends and improves the corresponding result of [17, 20, 21, 22].

The purpose of this paper to extends the results of Saluja and Nashine [16] for a finite family of asymptotically quasi-nonexpansive in the intermediate sense mappings.

For the sake of convenience, we first recall some definitions and notations.

In 1970, Takahashi [18] introduced the concept of convexity in a metric space and the properties of the space.

Definition 1.2([18]): Let (E, D) be a metric space and $I = [0,1]$. A mapping $W\colon E \times E \times I \to E$ is said to be a convex structure on E if for each $(x, y, \lambda) \in E \times E \times I$ and $u \in E$,

$$d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1 - \lambda) d(u, y).$$

E together with a convex structure W is called a *convex metric space*, denoted it by (E, d, W). A nonempty subset K of E is said to be *convex* if $W(x, y, \lambda) \in K$ for all $(x, y, \lambda) \in K \times K \times I$.

Remark 1.2: Every normed space is a convex metric space, where a convex structure $W(x, y, z; \alpha, \beta, \gamma) = \alpha x + \beta y + \gamma z$, for all $x, y, z \in E$ and $\alpha, \beta, \gamma \in I$ with $\alpha + \beta + \gamma = 1$. In fact,

$$\begin{aligned} d(u, W(x, y, z; \alpha, \beta, \gamma)) &= \|u - (\alpha x + \beta y + \gamma z)\| \\ &\leq \alpha \|u - x\| + \beta \|u - y\| + \gamma \|u - z\| \\ &= \alpha d(u, x) + \beta d(u, y) + \gamma d(u, z), \end{aligned} \tag{1.12}$$

for all $u \in E$. But there exists some convex metric spaces which can not be embedded into normed space.

Example 1.1: Let $X = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 > 0, x_2 > 0, x_3 > 0\}$. For $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in X$ and $\alpha, \beta, \gamma \in I$ with $\alpha + \beta + \gamma = 1$, we define a mapping $W\colon X^3 \times I^3 \to X$ by

$$W(x, y, z; \alpha, \beta, \gamma) = (\alpha x_1 + \beta y_1 + \gamma z_1, \alpha x_2 + \beta y_2 + \gamma z_2, \alpha x_3 + \beta y_3 + \gamma z_3),$$

and define a metric $d\colon X \times X \to [0, \infty)$ by

$$d(x, y) = |x_1y_1 + x_2y_2 + x_3y_3|.$$

Then we can show that (X, d, W) is a convex metric space, but it is not a normed space.

Definition 1.3: Let (E, d, W) be a convex metric space with a convex structure W and let $T_1, T_2, \ldots, T_N\colon X \to X$ be N asymptotically quasi-nonexpansive in the intermediate sense mappings. For any given $x_0 \in E$, the iteration process $\{x_n\}$ defined by

$$\begin{aligned}
x_1 &= W(x_0, T_1x_1, u_1; \alpha_1, \beta_1, \gamma_1),\\
x_2 &= W(x_1, T_2x_2, u_2; \alpha_2, \beta_2, \gamma_2),\\
&\vdots\\
x_N &= W(x_{N-1}, T_Nx_N, u_N; \alpha_N, \beta_N, \gamma_N),\\
x_{N+1} &= W(x_N, T_1^2x_{N+1}, u_{N+1}; \alpha_{N+1}, \beta_{N+1}, \gamma_{N+1}),\\
&\vdots\\
x_{2N} &= W(x_{2N-1}, T_N^2x_{2N}, u_{2N}; \alpha_{2N}, \beta_{2N}, \gamma_{2N}),\\
x_{2N+1} &= W(x_{2N}, T_1^3x_{2N+1}, u_{2N+1}; \alpha_{2N+1}, \beta_{2N+1}, \gamma_{2N+1}),\\
&\vdots
\end{aligned}$$

which can be written in the following compact form:

$$x_n = W(x_{n-1}, T^n_{n(modN)}x_n, u_n; \alpha_n, \beta_n, \gamma_n), \quad n \geq 1 \tag{1.13}$$

where $\{u_n\}$ is a bounded sequence in E, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are three sequences in $[0, 1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ for $n = 1, 2, \ldots$. Iteration process (1.13) is called the implicit iteration process with errors for a finite family of mappings $\{T_i\}_{i=1}^N$.

If $u_n = 0$ in (1.13) then,

$$x_n = W(x_{n-1}, T^n_{n(modN)}x_n; \alpha_n, \beta_n), \quad n \geq 1 \tag{1.14}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ be two sequences in in $[0, 1]$ such that $\alpha_n + \beta_n = 1$ for $n = 1, 2, \ldots$.

Iteration process (1.14) is called the implicit iteration process a finite family of mappings $\{T_i\}_{i=1}^N$.

In order to prove our main result of this paper, we need the following lemma.

Lemma 1.1(see [13]): Let $\{p_n\}$, $\{q_n\}$, $\{r_n\}$ be three nonnegative sequences of real numbers satisfying the following conditions:

$$p_{n+1} \leq (1+q_n)p_n + r_n, \quad n \geq 0, \quad \sum_{n=0}^{\infty} q_n < \infty, \quad \sum_{n=0}^{\infty} r_n < \infty. \tag{1.15}$$

Then

(1) $\lim_{n\to\infty} p_n$ exists.

(2) In addition, if $\liminf_{n\to\infty} p_n = 0$, then $\lim_{n\to\infty} p_n = 0$.

2 Main Results

Now we state and prove our main results of this paper.

Theorem 2.1: Let (E, d, W) be a complete convex metric space. Let $T_i \colon E \to E$ be a finite family of asymptotically quasi-nonexpansive in the intermediate sense mappings for $i = 1, 2, \dots, N$ such that $F = \cap_{i=1}^N F(T_i) \neq \emptyset$. Put

$$G_n \quad = \quad \max\left\{0, \sup_{p\in F,\ n\geq 1} \Big(d(T^n_{n(modN)}x_n, p) - d(x_n, p)\Big)\right\}. \tag{2.1}$$

Assume that $\sum_{n=1}^{\infty} G_n < \infty$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ and $\{\alpha_n\} \subset (s, 1-s)$ for some $s \in (0,1)$. Then the sequence $\{x_n\}$ defined by (1.13) has the following conclusions:

(1) $\lim_{n\to\infty} d(x_n, F)$ exists;

(2) the sequence $\{x_n\}$ converges strongly to a common fixed point p of the mappings $\{T_i\}_{i=1}^N$ if and only if

$$\liminf_{n\to\infty} d(x_n, F) = 0,$$

where $d(x, F) = \inf_{p\in F} d(x, p)$.

Proof: We divide the proof of Theorem 2.1 into two steps.

(I) First, we prove the conclusion (1).

For any $p \in F = \cap_{i=1}^{N} F(T_i)$, using (1.13) and (2.1), we have

$$\begin{aligned} d(x_n, p) &= d(W(x_{n-1}, T^n_{n(modN)} x_n, u_n; \alpha_n, \beta_n, \gamma_n), p) \\ &\leq \alpha_n d(x_{n-1}, p) + \beta_n d(T^n_{n(modN)} x_n, p) + \gamma_n d(u_n, p) \\ &\leq \alpha_n d(x_{n-1}, p) + \beta_n [d(x_n, p) + G_n] + \gamma_n d(u_n, p) \\ &\leq \alpha_n d(x_{n-1}, p) + \beta_n d(x_n, p) + \beta_n G_n + \gamma_n d(u_n, p) \\ &= \alpha_n d(x_{n-1}, p) + (1 - \alpha_n - \gamma_n) d(x_n, p) + \beta_n G_n + \gamma_n d(u_n, p) \\ &\leq \alpha_n d(x_{n-1}, p) + (1 - \alpha_n) d(x_n, p) + G_n + \gamma_n d(u_n, p), \end{aligned} \tag{2.2}$$

which on simplifying, we have

$$\begin{aligned} d(x_n, p) &\leq d(x_{n-1}, p) + \frac{1}{\alpha_n} G_n + \frac{\gamma_n}{\alpha_n} d(u_n, p) \\ &\leq d(x_{n-1}, p) + \frac{1}{\alpha_n} G_n + \frac{M}{\alpha_n} \gamma_n, \end{aligned} \tag{2.3}$$

where $M = \sup_{n \geq 1} d(u_n, p)$, since $\{u_n\}$ is bounded sequence in E.

Since $0 < s < \alpha_n < 1 - s < 1$, it follows from (2.3) that

$$d(x_n, p) \leq d(x_{n-1}, p) + \left\{ \frac{1}{s} G_n + \frac{M}{s} \gamma_n \right\}. \tag{2.4}$$

Since, by hypothesis,

$$\sum_{n=1}^{\infty} G_n < \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \gamma_n < \infty,$$

we have

$$\left\{ \frac{1}{s} \sum_{n=1}^{\infty} G_n + \frac{M}{s} \sum_{n=1}^{\infty} \gamma_n \right\} < \infty.$$

In (2.4) taking infimum over all $p \in F$, we have

$$d(x_n, F) \leq d(x_{n-1}, F) + \left\{ \frac{1}{s} G_n + \frac{M}{s} \gamma_n \right\}. \tag{2.5}$$

It follows from Lemma 1.1 that

$$\lim_{n\to\infty} d(x_n, F) \quad \text{exists.} \tag{2.6}$$

The conclusion (1) is proved.

(II) The proof of conclusion (2).

Necessity

If $\{x_n\}$ converges strongly to some common fixed point $p \in F$, then, we have

$$\liminf_{n\to\infty} d(x_n, F) = 0. \tag{2.7}$$

Sufficiency

If $\liminf_{n\to\infty} d(x_n, F) = 0$, then from Lemma 1.1(2), we have $\lim_{n\to\infty}$ $d(x_n, F) = 0$.

Thus for any $\varepsilon > 0$ there exists a positive integer N_1 such that for $n \geq N_1$,

$$d(x_n, F) < \frac{\varepsilon}{6}. \tag{2.8}$$

Again since $\sum_{n=1}^{\infty} G_n < \infty$ and $\sum_{n=1}^{\infty} \gamma_n < \infty$ imply that there exist positive integers N_2 and N_3 such that

$$\sum_{j=n}^{\infty} G_j < \frac{s\varepsilon}{6}, \quad \forall n \geq N_2 \tag{2.9}$$

and

$$\sum_{j=n}^{\infty} \gamma_j < \frac{s\varepsilon}{6M}, \quad \forall n \geq N_3 \tag{2.10}$$

Let $N = \max\{N_1, N_2, N_3\}$. It follows from (2.4), that

$$d(x_n, p) \quad \leq \quad d(x_{n-1}, p) + \frac{1}{s} G_n + \frac{M}{s} \gamma_n. \tag{2.11}$$

Now, for each $m, n \geq N$, we have

$$
\begin{aligned}
d(x_n, x_m) &\leq d(x_n, p) + d(x_m, p) \\
&\leq d(x_N, p) + \frac{1}{s} \sum_{j=N+1}^{n} G_j + \frac{M}{s} \sum_{j=N+1}^{n} \gamma_j \\
&\quad + d(x_N, p) + \frac{1}{s} \sum_{j=N+1}^{n} G_j + \frac{M}{s} \sum_{j=N+1}^{n} \gamma_j \\
&\leq 2d(x_N, p) + \frac{2}{s} \sum_{j=N+1}^{n} G_j + \frac{2M}{s} \sum_{j=N+1}^{n} \gamma_j \\
&< 2.\frac{\varepsilon}{6} + \frac{2}{s}.\frac{s\varepsilon}{6} + \frac{2M}{s}.\frac{s\varepsilon}{6M} \\
&< \varepsilon. \qquad (2.12)
\end{aligned}
$$

This implies that $\{x_n\}$ is a Cauchy sequence in E. Thus, the completeness of E implies that $\{x_n\}$ must be convergent. Assume that $\lim_{n\to\infty} x_n = p^*$. Now, we have to show that p^* is a common fixed point of the mappings $\{T_i\}_{i=1}^N$. Indeed, we know that the set $F = \cap_{i=1}^N F(T_i)$ is closed. From the continuity of $d(x, F) = 0$ with $\lim_{n\to\infty} d(x_n, F) = 0$ and $\lim_{n\to\infty} x_n = p^*$, we get

$$d(p^*, F) = 0, \qquad (2.13)$$

and so $p^* \in F$, that is, p^* is a common fixed point of the mappings $\{T_i\}_{i=1}^N$. This completes the proof.

If $u_n = 0$, in Theorem 2.1, we can easily obtain the following theorem.

Theorem 2.2: Let (E, d, W) be a complete convex metric space. Let $T_i\colon E \to E$ be a finite family of asymptotically quasi-nonexpansive in the intermediate sense mappings for $i = 1, 2, \ldots, N$ such that $F = \cap_{i=1}^N F(T_i) \neq \emptyset$. Put

$$G_n = \max\left\{0, \sup_{p\in F,\ n\geq 1} \left(d(T^n_{n(modN)} x_n, p) - d(x_n, p)\right)\right\}.$$

Assume that $\sum_{n=1}^\infty G_n < \infty$ and $\{\alpha_n\} \subset (s, 1-s)$ for some $s \in (0, 1)$. Then the sequence $\{x_n\}$ defined by (1.14) converges strongly to a common fixed point p of the mappings $\{T_i\}_{i=1}^N$ if and only if

$$\liminf_{n\to\infty} d(x_n, F) = 0.$$

Remark 2.1: Our results extend and improve the corresponding results of Sun [17], Wittmann [20] and Xu and Ori [21] to the case of more general class of non-expansive and asymptotically quasi-nonexpansive mappings and implicit iteration process with errors considered in this paper.

Remark 2.2: Our results also extend the corresponding results of Kim et. al. [10] and Saluja and Nashine [16] to the case of more general class of asymptotically quasi-nonexpansive mappings considered in this paper.

Remark 2.3: Our results also extend the corresponding results of Zhou and Chang [22] to the case of more general class of asymptotically nonexpansive mappings considered in this paper.

Remark 2.4: The main result of this paper is also an extension and improvement of the well-known corresponding results in [1]-[9] and [12]-[15].

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