Functional Analysis,
Approximation and
Computation
4:1 (2012), 1–7

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/faac

$\mathcal{I}^{\mathcal{K}}$- convergence in 2- normed spaces

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Abstract. In this paper, we introduce the concept of $\mathcal{I}^{\mathcal{K}}-$convergence of sequences in 2–normed spaces. This notion can be regard as an extension of $\mathcal{I}$-convergence of sequences in 2–normed spaces.

1. Introduction

The idea of $\mathcal{I}-$convergence was informally introduced by Kostyrko et al (2001) and also independently by Nuray and Ruckle (2000), as a generalization of statistical convergence. Ideal convergence provides a general framework to study the properties of various types of convergence.
There are several works in recent years on $\mathcal{I}-$convergence of sequences (see [4,9,21]).

The notion of linear 2–normed spaces has been investigated by Gâhler in1960's [7,8] and has been developed extensively in different subjects by others [2,11,18,23]. It seems therefore reasonable to investigate the concepts of $\mathcal{I}$ and $\mathcal{I}^*-$convergence in 2–normed spaces. Throughout this paper $\mathbb{N}$ will denote the set of positive integers.

Definition 1.1. *Let X be a real linear space of dimension greater than 1, and* $\|.,.\|$ *be a non-negative real–valued function on* $X \times X$ *satisfying the following conditions:*

G1)$\|x, y\| = 0$ *if and only if x and y are linearly dependent vectors,*

G2) $\|x, y\| = \|y, x\|$ *for all x,y in X,*

G3)$\|\alpha x, y\| = |\alpha|\|x, y\|$ *where* α *is real number,*

G4)$\|x + y, z\| \leq \|x, z\| + \|y, z\|$ *for all x, y, z in X.*

$\|.,.\|$ *is called a 2–norm on X and the pair* $(X, \|.,.\|)$ *is called a linear 2–normed space.*

Every linear 2–normed space $(X, \|.,.\|)$ *of dimension different from one is a locally convex topological vector space. In fact, for a fixed* $b \in X$, $p_b(x) = \|x, b\|, x \in X$, *is a seminorm and the family* $P = \{p_b : b \in X\}$*of seminorms generates a locally convex topology on X. In addition, we have the following properties:*

1)$\|.,.\|$ *is nonnegative.*

2010 *Mathematics Subject Classification*. Primary 40A05; Secondary 40C99, 40A99, 54A20.
Keywords. $I-$ convergence; I^*- convergence; Filter; Double sequences; 2–normed space.
Received: January 2, 2012; Accepted: January 24, 2012
Communicated by Dragan S. Djordjević
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2) $\|x, y\| = \|x, y + \alpha x\|$
3) $\|x - y, y - z\| = \|x - y, x - z\|$
for all scalars α *and all* $x, y, z \in X$. *Some of the basic properties of 2–norms studied in [18].*
As an example of a 2-normed space we may take $X = \mathbb{R}^2$ *being equipped with the 2-norm* $\|x, y\|$:= *the area of the parallelogram spanned by the vectors x and y, which may be given clearly by the formula*

$$\|x, y\| = |x_1 y_2 - x_2 y_1| \qquad , \qquad x = (x_1, x_2) \qquad y = (y_1, y_2).$$

Given a 2-normed space $(X, \|.,.\|)$. *One can derive a topology for it via the following definition of the limit of a sequence: A sequence* $(x_n)_{n\in\mathbb{N}}$ *in X is said to be convergent to x in X if* $\lim_{n\to\infty} \|x_n - x, z\| = 0$ *for all* $z \in X$. *This can be written by the formula:*

$$(\forall z \in Y)(\forall \epsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0) \qquad \|x_n - x, z\| < \epsilon.$$

We write it as $\quad x_n \xrightarrow{\|.,.\|_X} x.$

2. Preliminary Notes

We recall the following definition, where Y represents an arbitrary set.

Definition 2.1. *A family* $\mathcal{I} \subseteq \mathcal{P}(Y)$ *of subsets a nonempty set Y is said to be an ideal in Y if:*
i) $\emptyset \in \mathcal{I}$,
ii) $A, B \in \mathcal{I}$ *implies* $A \bigcup B \in \mathcal{I}$,
iii) $A \in \mathcal{I}, B \subseteq A$ *implies* $B \in \mathcal{I}$,
$\mathcal{I}$ *is called a proper ideal if* $Y \notin \mathcal{I}$ *and* $\mathcal{I}$ *is not proper ideal if* $\mathcal{I} = \mathcal{P}(Y)$.

Definition 2.2. *The ideal of all finite subsets of a given set Y is called Fin.*

Definition 2.3. *Let* $Y \neq \emptyset$. *A non empty family F of subsets of Y is said to be a filter in Y provided:*
i) $\emptyset \in F$.
ii) $A, B \in F$ *implies* $A \cap B \in F$.
iii)$A \in F, A \subseteq B$ *implies* $B \in F$.
If $\mathcal{I}$ *is a nontrivial ideal in* Y,$Y \neq \emptyset$, *then the class*

$$F(\mathcal{I}) = \{M \subset Y : (\exists A \in \mathcal{I})M = Y - A\}$$

is a filter on Y, called the filter associated with $\mathcal{I}$.

Definition 2.4. *A nontrivial ideal* $\mathcal{I}$ *in Y is called admissible if* $\{x\} \in \mathcal{I}$ *for each* $x \in Y$.

Definition 2.5. *A nontrivial ideal* $\mathcal{I}$ *in* $\mathbb{N} \times \mathbb{N}$ *is called strongly admissible if* $\{i\} \times \mathbb{N}$ *and* $\mathbb{N} \times \{i\}$ *belong to* $\mathcal{I}$ *for all* $i \in \mathbb{N}$.
It is evident that a strongly admissible ideal is admissible also.
Let
$\mathcal{I}_0$=$\{A \subset \mathbb{N} \times \mathbb{N} : (\exists m(A) \in \mathbb{N})(i, j \geq m(A) \Rightarrow (i, j) \in \mathbb{N} \times \mathbb{N} - A)\}$.
Then $\mathcal{I}_0$ *is a nontrivial strongly admissible ideal and clearly an ideal* $\mathcal{I}$ *is strongly admissible if and only if* $\mathcal{I}_0 \subseteq \mathcal{I}$ *[3].*

Definition 2.6. *Let* $Y \neq \emptyset$ *be a set and K be an ideal on Y. Let* $M \subseteq Y$, $M \neq \emptyset$. $K|M := \{A \cap B : A \in K\}$ *is called trace of K on M and* $K|M$ *is an ideal on Y.*

Definition 2.7. *Let* $\mathcal{I} \subseteq \mathcal{P}(\mathbb{N})$ *be a nontrivial ideal in* $\mathbb{N}$. *The sequence* $(x_n)_{n\in\mathbb{N}}$ *in X is said to be* $\mathcal{I}$*-convergent to* $x \in X$, *if for each* $\epsilon > 0$, *the set* $A(\epsilon)$=$\{n \in \mathbb{N} : \|x_n - x\| \geq \epsilon\}$ *belongs to* $\mathcal{I}$ *[1,12,14].*

3. $\mathcal{I}^{\mathcal{K}}$- convergence in 2-normed spaces

In [9,14,21], the concepts of $\mathcal{I}$ and $\mathcal{I}^*$-convergence introduced in 2-normed space. We extend this concepts and introduce the $\mathcal{I}^{\mathcal{K}}$-convergence for sequences in 2-normed spaces.
First, we introduce the definition of $\mathcal{I}$ and $\mathcal{I}^*$-convergence by the other manner.

Definition 3.1. *Let $(X, \|.,.\|)$ be a 2–normed space and $\mathcal{I}$ be an ideal on a set A. The function $f : A \to X$ is said to be $\mathcal{I}$-convergent to $x \in X$ if for all non zero z in X and for all $\varepsilon > 0$, we have:*

$$A(\epsilon) = \{a \in \mathbb{A} : \|f(a) - x, z\| \geq \epsilon\} \in \mathcal{I}.$$

We write it as

$$\mathcal{I} - \lim f = x.$$

Remark 3.2. If $A = N$, we obtain the usual definition $\mathcal{I}$-convergent of sequence $(x_n)_{n\in\mathbb{N}}$ to $x \in X$ in 2-normed space X [9].

Lemma 3.3. Let X, Y be two 2–normed spaces and let A be a non empty set and I, I_1, I_2 be ideals on A. Then
i) if I is not proper ideal, then every function $f : A \to X$ is $\mathcal{I}-$convergent to each point of X.
ii) If $I_1 \subset I_2$, then for every function $f : A \to X$, we have

$$I_1 - \lim f = x \qquad \Rightarrow \qquad I_2 - \lim f = x.$$

Proof: i) Let x be arbitrary element of X, then

$$(\forall z \in X)(\forall \varepsilon > 0)A(\epsilon) = \{a \in \mathbb{A} : \|f(a) - x, z\| \geq \epsilon\} \in P(S) = I.$$

ii) Let $I_1 \subset I_2$, $I_1 - \lim f = x$. Then we have

$$(\forall z \in X)(\forall \varepsilon > 0)A(\epsilon) = \{a \in \mathbb{A} : \|f(a) - x, z\| \geq \epsilon\} \in I_1 \subset I_2.$$

Hence $I_2 - \lim f = x$.

Definition 3.4. *Let $(X, \|.,.\|)$ be a 2–normed space and $\mathcal{I}$ be an ideal on $\mathbb{N}$. The sequence $(x_n)_{n\in\mathbb{N}}$ in X is said to be $\mathcal{I}^*$-convergent to a point $l \in X$, if there exists a set $M \in \mathcal{F}(\mathcal{I})$ such that*

$$x_n \xrightarrow{\|.,.\|_X} l \qquad (on \quad M).$$

We write it as

$$\mathcal{I}^* - \lim x_n = l.$$

We introduce the definition of $\mathcal{I}^{\mathcal{K}}$-convergence, we simply replace the ideal Fin by an arbitrary ideal on the set A.

Definition 3.5. *Let $(X, \|.,.\|)$ be 2–normed space and let $\mathcal{K}$ and $\mathcal{I}$ be ideals on $\mathbb{N}$, we say that a sequence $(x_n)_{n\in\mathbb{N}}$ in X is $\mathcal{I}^{\mathcal{K}}$- convergent to $x \in X$ if:*
there exists a set $M \in \mathcal{F}(\mathcal{I})$ and the sequence $(y_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & if\, n \in M \\ x & if\, n \notin M \end{cases}$$

such that $\mathcal{K} - \lim y_n = x$.
We write it as

$$\mathcal{I}^{\mathcal{K}} - \lim x_n = x.$$

Remark 3.6. The definition of $\mathcal{I}^{\mathcal{K}}-$ convergence can be reformulated in the form of decomposition. A sequence $(x_n)_{n\in\mathbb{N}}$ is $\mathcal{I}^{\mathcal{K}}-$ convergence if and only if $(x_n)_{n\in\mathbb{N}} = (y_n)_{n\in\mathbb{N}} + (z_n)_{n\in\mathbb{N}}$, where $(y_n)_{n\in\mathbb{N}}$ is $\mathcal{K}$-convergent and $(z_n)_{n\in\mathbb{N}}$ is non-zero only on a set from $\mathcal{I}$.

Remark 3.7. Another definition of $\mathcal{I}^{\mathcal{K}}$- convergence can be the form below: The sequence $(x_n)_{n\in\mathbb{N}}$ is $\mathcal{I}^{\mathcal{K}}$-convergent if there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $x_n|_M = (x_n)_{n\in M}$ is $K|M$ -convergent to x. These two definitions are equivalent but definition 3.5 is simpler.

We give some examples of ideals and corresponding $\mathcal{I}^{\mathcal{K}}-$ convergence.

Example 3.8. (i) Put $\mathcal{I}_\circ = \mathcal{K}_\circ = \{\emptyset\}$. $\mathcal{I}_\circ$ is the minimal ideal in $\mathbb{N}$.
A sequence $(x_n)_{n\in\mathbb{N}}$ is $\mathcal{I}_\circ^{\mathcal{K}_\circ}$- convergent if and only if it is constant.
(ii) Let $\emptyset \neq M \subset \mathbb{N}$, $M \neq \mathbb{N}$. Put $\mathcal{K} = P(M)$, i.e. $\mathcal{K}$ is a proper ideal in $\mathbb{N}$. Let $\mathcal{I} = \{\emptyset\}$. A sequence $(x_n)_{n\in\mathbb{N}}$ is $\mathcal{I}^{\mathcal{K}}$- convergent if and only if it is constant on $\mathbb{N} \setminus M$.
(iii) Let $\mathcal{K}$ be an admissible ideal in $\mathbb{N}$ and $\mathcal{I}$ be a arbitrary ideal.
A sequence $(x_n)_{n\in\mathbb{N}}$ is $\mathcal{I}^{\mathcal{K}}-$ convergent if there exists a set $M \in \mathcal{F}(\mathcal{I})$ and the sequence $(y_n)_{n\in\mathbb{N}}$ given by definition such that the sequence y_n is the usual converges.

Corollary 3.9. Let $(X, \|.,.\|)$ be a 2–normed space, and let $(x_n)_{n\in\mathbb{N}}$ be a convergent sequence in X and $l_1, l_2 \in X$. If $\mathcal{I}^{\mathcal{K}} - \lim x_n = l_1$ and $\mathcal{I}^{\mathcal{K}} - \lim x_n = l_2$ then $l_1 = l_2$.
Proof: Suppose $l_1 \neq l_2$. Hence there exists $z \in X$ such that $l_1 - l_2 (\neq 0)$ and z are linearly independent. Put

$$\|l_1 - l_2, z\| = 2\varepsilon, \qquad with \qquad \varepsilon > 0.$$

Since $\mathcal{I}^{\mathcal{K}} - \lim x_n = l_1$. By the definition, there exists $M_1, M_2 \in \mathcal{F}(\mathcal{I})$ such that the sequences $(y_n)_{n\in\mathbb{N}}$, $(z_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & \text{if } n \in M_1 \\ l_1 & \text{if } n \notin M_1 \end{cases} \qquad z_n = \begin{cases} x_n & \text{if } n \in M_2 \\ l_2 & \text{if } n \notin M_2 \end{cases}$$

have the following properties

$$(\forall\varepsilon > 0)(\forall z \in X) \qquad \{n \in \mathbb{N} : \|y_n - l_1, z\| \geq \epsilon\} \in \mathcal{K}$$

$$(\forall\varepsilon > 0)(\forall z \in X) \qquad \{n \in \mathbb{N} : \|z_n - l_2, z\| \geq \epsilon\} \in \mathcal{K}.$$

Put $M = M_1 \cap M_2$. We have

$$2\varepsilon = \|l_1 - x_n + x_n - l_2, z\| \leq \|x_n - l_1, z\| + \|x_n - l_2, z\| = \|y_n - l_1, z\| + \|z_n - l_2, z\|$$

Therefor $\{n \in M : \|z_n - l_2, z\| < \varepsilon\} \subseteq \{n \in M : \|y_n - l_1, z\| \geq \varepsilon\} \in \mathcal{K}$. Hence $\{n \in M : \|z_n - l_2, z\| < \varepsilon\} \in \mathcal{K}$ that is contradict with $\mathcal{I} \neq \phi$.

Corollary 3.10. If $(x_n)_{n\in\mathbb{N}}$, $(y_n)_{n\in\mathbb{N}}$ be sequences in 2-normed space $(X, \|.,.\|)$ and $\mathcal{I}^{\mathcal{K}} - \lim x_n = a$, $\mathcal{I}^{\mathcal{K}} - \lim y_n = b$, then
i)$\mathcal{I}^{\mathcal{K}} - \lim x_n + y_n = a + b$,
ii)$\mathcal{I}^{\mathcal{K}} - \lim \alpha x_n = \alpha a$.

Proof: (i) Let $\mathcal{I}^{\mathcal{K}} - \lim x_n = a$, $\mathcal{I}^{\mathcal{K}} - \lim y_n = b$. By the definition, there exist $M_1, M_2 \in \mathcal{F}(\mathcal{I})$ such that the sequences $(z_n)_{n\in\mathbb{N}}$, $(t_n)_{n\in\mathbb{N}}$ given by

$$z_n = \begin{cases} x_n & \text{if } n \in M_1 \\ a & \text{if } n \notin M_1 \end{cases} \qquad t_n = \begin{cases} y_n & \text{if } n \in M_2 \\ b & \text{if } n \notin M_2 \end{cases}$$

has the following properties $\mathcal{K} - \lim z_n = a$, and $\mathcal{K} - \lim t_n = b$.
Now, we put $M = M_1 \cap M_2 \in \mathcal{F}(\mathcal{I})$ and define the sequence $\{p_n\}$ by

$$p_n = \begin{cases} x_n + y_n & \text{if } n \in M \\ a + b & \text{if } n \notin M, \end{cases}$$

we have $\mathcal{K} - \lim z_n + t_n = \mathcal{K} - \lim z_n + \mathcal{K} - \lim t_n = a + b$ (see [21]). By the definition, $\mathcal{I}^{\mathcal{K}} - \lim x_n + y_n = a + b$.
The proof of (ii) is similar to (i).

In the next lemma, we show easily from the definitions that $\mathcal{K}$- convergence implies the $\mathcal{I}^{\mathcal{K}}$- convergence.

Lemma 3.11. Let $\mathcal{K}$ and $\mathcal{I}$ be ideals on a set $\mathbb{N}$. If $(X, \|.,.\|)$ be a 2-normed space and $(x_n)_{n\in\mathbb{N}}$ be a sequence in X such that $\mathcal{K} - \lim x_n = x$, then
$\mathcal{I}^{\mathcal{K}} - \lim x_n = x$.

Lemma 3.12. Let $(X, \|.,.\|)$ be a 2–normed space and let $\mathcal{I}$, $\mathcal{I}_1$, $\mathcal{I}_2$, $\mathcal{K}$, $\mathcal{K}_1$ and $\mathcal{K}_2$ be ideals on a set $\mathbb{N}$ such that $\mathcal{I}_1 \subset \mathcal{I}_2$ and $\mathcal{K}_1 \subset \mathcal{K}_2$. Then for every sequence $(x_n)_{n\in\mathbb{N}}$ in X, we have:

$$i) \quad \mathcal{I}_1^{\mathcal{K}} - \lim x_n = x \quad \textit{implies} \quad \mathcal{I}_2^{\mathcal{K}} - \lim x_n = x,$$

$$ii) \quad \mathcal{I}^{\mathcal{K}_1} - \lim x_n = x \quad \textit{implies} \quad \mathcal{I}^{\mathcal{K}_2} - \lim x_n = x.$$

Proof: i) Suppose $\mathcal{I}_1^{\mathcal{K}} - \lim x_n = x$, by definition, there exists $M \in \mathcal{F}(\mathcal{I}_1)$ such that the sequence $(y_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & \text{if } n \in M \\ x & \text{if } n \notin M \end{cases}$$

has the following condition

$$(\forall \varepsilon > 0)(\forall z \in X) A(\epsilon) = \{n \in \mathbb{N} : \|y_n - x, z\| \geq \epsilon\} \in \mathcal{K}.$$

On the other hand since $\mathcal{I}_1 \subset \mathcal{I}_2$, then $M \in \mathcal{F}(\mathcal{I}_1) \subset \mathcal{F}(\mathcal{I}_2)$ and by the definition (3.5), $\mathcal{I}_2^{\mathcal{K}} - \lim x_n = x$.
ii) Let $\mathcal{I}^{\mathcal{K}_1} - \lim x_n = x$. By definition, there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $(y_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & \text{if } n \in M \\ x & \text{if } n \notin M \end{cases}$$

has the following property

$$(\forall \varepsilon > 0)(\forall z \in X) A(\epsilon) = \{n \in \mathbb{N} : \|y_n - x, z\| \geq \epsilon\} \in \mathcal{K}_1 \subset \mathcal{K}_2.$$

Hence $A(\epsilon) \in \mathcal{K}_2$ and the proof is complete.

In the next theorem, we prove the relationship between the $\mathcal{I}$-convergence and $\mathcal{I}^{\mathcal{K}}$- convergence.

Theorem 3.1: Let $(X, \|.,.\|)$ be a 2-normed space and let $\mathcal{K}$ and let $\mathcal{I}$ be two ideals in $\mathbb{N}$. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in X.
i) If $\mathcal{I}^{\mathcal{K}} - \lim x_n = x$ *implies* $\mathcal{I} - \lim x_n = x$ holds for some $x \in X$, which has at least one neighborhood different from X, then $\mathcal{K} \subseteq \mathcal{I}$.
ii) If $\mathcal{K} \subseteq \mathcal{I}$ then ($\mathcal{I}^{\mathcal{K}} - \lim x_n = x$ *implies* $\mathcal{I} - \lim x_n = x$).

Proof:
i) Suppose that $\mathcal{K}$ is not subset of $\mathcal{I}$. Then there exists a set $A \in \mathcal{K}$ such that $A \notin \mathcal{I}$. Let $x \in X$ has a

neighborhood $U \subset X$ such that $U \neq X$ and $y \in X \setminus U$.
We define a sequence $\{y_n\}$ on X by

$$y_n = \begin{cases} y & \text{if } n \in A \\ x & \text{if } n \notin A. \end{cases}$$

Clearly, $\mathcal{K} - \lim x_n = x$. Thus by Lemma 3.11, we obtain $\mathcal{I}^{\mathcal{K}} - \lim x_n = x$. By the definition, $\{n \in \mathbb{N} : \|y_n - x, z\| \geq \epsilon\} = A \notin \mathcal{I}$. Hence the sequence $(x_n)_{n\in\mathbb{N}}$ is not $\mathcal{I}$-convergent to x.
ii) Let $(X, \|.,.\|)$ be a 2-normed space and $x \in X$ and let $(x_n)_{n\in\mathbb{N}}$ be a sequence on X. Let $\mathcal{K} \subseteq \mathcal{I}$, $\mathcal{I}^{\mathcal{K}} - \lim x_n = x$. By the definition of $\mathcal{I}^{\mathcal{K}}$- convergence, there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $(y_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & \text{if } n \in M \\ x & \text{if } n \notin M \end{cases}$$

has the condition

$$(\forall \varepsilon > 0)(\forall z \in X) A(\epsilon) = \{n \in \mathbb{N} : \|y_n - x, z\| \geq \epsilon\}$$
$$= \{n \in M : \|x_n - x, z\| \geq \epsilon\}.$$

Hence $A(\epsilon) \cap M \in \mathcal{K} \subseteq \mathcal{I}$. Consequently,

$$\{n \in \mathbb{N} : \|x_n - x, z\| \geq \epsilon\} \subseteq (X \setminus M) \cup (A(\epsilon) \cap M) \in \mathcal{I}$$

and thus $\mathcal{I} - \lim x_n = x$.

Example 3.13. Let $\mathbb{N} = \cup_{i=1}^{\infty} D_i$ be a decomposition of $\mathbb{N}$ (i.e. $D_i \cap D_j \neq \emptyset$). Assume that $D_i \quad (i = 1, 2, \cdots)$ are infinite sets (we can choose $D_i = \{3^{i-1}(2t) : t \in \mathbb{N}\}$ for $i = 1, 2, \cdots$). Denote by $\mathcal{I}$ the class of all $A \subseteq \mathbb{N}$ such that A intersects only a finite numbers of D_i. Let $\mathcal{K}$ be the family of all finite subsets of $\mathbb{N}$. Define $(x_n)_{n\in\mathbb{N}}$ as follows: For $n \in D_i$, we put $x_n = \frac{1}{i} \quad (i = 1, 2, \cdots)$. Obviously that $\mathcal{I} - \lim x_n = 0$. Now we show that $\mathcal{I}^{\mathcal{K}} - \lim x_n \neq 0$. Assume that $\mathcal{I}^{\mathcal{K}} - \lim x_n = 0$. Then there exists $M \in \mathcal{F}(\mathcal{I})$ such that the sequence $(y_n)_{n\in\mathbb{N}}$ given by

$$y_n = \begin{cases} x_n & \text{if } n \in M \\ 0 & \text{if } n \notin M, \end{cases}$$

satisfying $\mathcal{K} - \lim y_n = 0$, i.e.$\lim y_n = 0$. Since $M \in \mathcal{F}(\mathcal{I})$, then there exists $A \in \mathcal{I}$ such that $M = \mathbb{N} \setminus A$. By the definition of $\mathcal{I}$, there exists an $l \in \mathbb{N}$ such that

$$A \subset D_1 \cup \cdots \cup D_l.$$

Then M contains the set D_{l+1} and $x_n = \frac{1}{l+1}$ for infinitely many n's in M. This contradicts

$$\lim_{\substack{n\to\infty \\ (n\in M)}} y_n = \lim_{n\to\infty} x_n = 0.$$

The concept of $\mathcal{I}_2$ and $\mathcal{I}_2^*$-convergence of double sequences in 2-normed spaces was introduced in [19,21]. Now, we show that $\mathcal{I}^{\mathcal{K}}$-convergence is a correct the generalization of $\mathcal{I}_2^*$-convergence for double sequences in 2-normed spaces.

Definition 3.14. *Let $x = (x_{jk})_{j,k\in\mathbb{N}}$ be a double sequence in 2-normed space $(X, \|.,.\|)$. A double sequence x=$(x_{jk})_{j,k\in\mathbb{N}}$ is said to be convergent to $l \in X$* in Pringsheim's sense *if*
$(\forall z \in X)(\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall j, k \geq N) \qquad \|x_{jk} - l, z\| < \varepsilon$.
We write it as

$$x_{jk} \xrightarrow{\|.,.\|_X} l.$$

Definition 3.15. *A double sequence* x=$(x_{jk})_{j,k\in\mathbb{N}}$ *in 2-normed space* $(X, \|.,.\|)$ *is said to be* $\mathcal{I}_2$*-convergent to* $l \in X$*, if for all* $\varepsilon > 0$ *and nonzero* $z \in X$*,*

$$A(\varepsilon) = \{(j,k) : \|x_{jk} - l, z\| \geq \varepsilon\} \in \mathcal{I}_2.$$

In this case we write it as

$$\mathcal{I}_2 - \lim_{j,k} x_{jk} = l.$$

The Pringsheim's ideal $\mathcal{I}_2$ on $\mathbb{N} \times \mathbb{N}$ whose dual filter $\mathcal{F}(\mathcal{I}_2)$ is given by the filter base

$$\mathcal{B}_2 = \{[n,\infty] \times [n,\infty]; n \in \mathbb{N}\}.$$

Definition 3.16. *A double sequence* x=$(x_{jk})_{j,k\in\mathbb{N}}$ *in 2-normed space* $(X, \|.,.\|)$ *is said to be* $\mathcal{I}_2^*$*-convergent to* $l \in X$*, if there exists a set* $M \in \mathcal{F}(\mathcal{I})$ *(i.e.*$N \times N \backslash M \in \mathcal{I}$*) such that* $\lim_{m,n} x_{mn}$=l $(m,n) \in M$ *and we write it as*

$$\mathcal{I}_2^* - \lim_{j,k} x_{jk} = l.$$

Remark 3.17. The $\mathcal{I}^*$-convergence of double sequences is the same as $\mathcal{I}^{\mathcal{I}_2}$-convergence in $\mathbb{N} \times \mathbb{N}$. Hence the notion of the $\mathcal{I}^{\mathcal{K}}-$convergence is a correct generalization of $\mathcal{I}$ and $\mathcal{I}^*$-convergence.

Acknowledgements

The authors would like to thank Professor Martin Sleziak for the valuable comments on this manuscript.

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