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Porosity and the weighted L^p-spaces

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Abstract. Let G be a locally compact group, ω be a weight function on G and $1 < p, q < \infty$. Recently, it has been introduced some important $\sigma - c-$lower porous subsets $L^p(G) \times L^q(G)$, where $1/p + 1/q < 1$. Using these achievements, almost all available results connected to the existence of convolution of two functions belonging to $L^p(G)$ are obtained. In the present work these results will be extended for the weighted case. In fact for $2 < p < \infty$, some important $\sigma - c-$lower porous subsets $L^p(G,\omega) \times L^p(G,\omega)$ are introduced.

1. Introduction and Preliminaries

Throughout the paper, let G be a locally compact group with a fixed left Haar measure λ and $\omega : G \to (0, \infty)$ be a weight function on G; that is, a Borel measurable function on G. The weight function ω is called submultiplicative if

$$\omega(xy) \leq \omega(x)\omega(y),$$

for all $x, y \in G$. We say ω is of moderate growth if

$$ess \sup_{y \in G} \frac{\omega(xy)}{\omega(y)} < \infty, \tag{1.1}$$

for all $x \in G$. For $1 \leq p < \infty$, the space $L^p(G,\omega)$ with respect to λ is the set of all complex valued measurable functions f on G such that $f\omega \in L^p(G)$, as defined in [10]. Let us remark that

$$\|f\|_{p,\omega} := \left(\int_G |f(x)|^p \omega(x)^p d\lambda(x) \right)^{1/p} \qquad (f \in L^p(G,\omega)),$$

defines a norm on $L^p(G,\omega)$ under which it a Banach space. We denote this space by $\ell^p(G,\omega)$ when G is discrete.

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For measurable functions f and g on G, the convolution

$$(f * g)(x) = \int_G f(y)\, g(y^{-1}x)\, d\lambda(y)$$

is defined at each point $x \in G$ for which the function $y \mapsto f(y)\, g(y^{-1}x)$ is λ-integrable. If $f * g(x)$ is defined and finite for $\lambda-$almost all $x \in G$, then we say that the convolution of functions $f * g$ exists as a function. The convolution $f * g$ does not necessarily exist for all measurable functions f and g. So, it would be interesting to know does $f * g$ exist for all functions f and g in a space X of measurable functions on G. If this is the case, then it is desirable to study the closedness of X under the convolution. It is well-known that $L^1(G)$ is always closed under the convolution. Saeki [13] proved that, for $1 < p < \infty$, the space $L^p(G)$ is closed under the convolution if and only if G is compact. But the convolution of elements in $L^p(G)$ even does not exist in general. Several authors have been studied the existence of convolution on certain function spaces. In fact on a locally compact non-compact group G, the space $L^p(G)$ for $2 < p < \infty$, contains functions f and g whose convolution is infinite on a set of positive measure; see [12] and also [1].

In a recent work, this result was strengthened by Glab and Strobin [8] and they proved a quantitative version of this result. Indeed, for $1 < p, q < \infty$, they considered the product $L^p(G) \times L^q(G)$ as a Banach space with a norm defined as the maximum of norms of coordinates, that is

$$\|(f, g)\| = \max\{\|f\|_p, \|g\|_q\}.$$

As a main result, they showed that if $1/p + 1/q < 1$ then for each compact subset K of locally compact non-compact group G, the set E_K of pairs $(f, g) \in L^p(G) \times L^q(G)$ for which $f * g$ is well-defined at some point of K (i. e. $f * g(x)$ is finite or equal to ∞ or $-\infty$), satisfies a porosity condition as the following: every ball about a point of E_K contains balls that are disjoint from it. Such sets are nowhere dense and thus if $2 < p < \infty$ and G is $\sigma-$compact then the pairs of functions whose convolution is nowhere defined is residual in $L^p(G) \times L^q(G)$. Then [1, Theorem 1.1] can immediately be obtained by using this result. See also [5], as a work in this direction.

Before stating the aim of this paper, we will briefly describe some notions of porosity from [14]. Let X be a metric space. For positive number R, the open ball with a radius R centered at a point $x \in X$ will be denoted by $B(x; R)$. Let $c \in (0, 1]$. We say that $M \subseteq X$ is $c-$lower porous if for each $x \in M$,

$$\liminf_{R \to 0^+} \frac{\gamma(x, M, R)}{R} \geq \frac{c}{2},$$

where

$$\gamma(x, M, R) = \sup\{r \geq 0 : \exists x \in X, B(z, r) \subseteq B(x; R) \setminus M\}.$$

Equivalently, M is $c-$lower porous if and only if for each $x \in M$ and $0 < \beta < c/2$, there exits $R_0 > 0$ and $z \in X$ such that for each $0 < R < R_0$,

$$B(z, \beta R) \subseteq B(x; R) \setminus M;$$

see [14]. If M is a countable union of $c-$lower porous sets then we say that M is $\sigma - c-$lower porous. It can be easily seen that the $c-$lower porosity implies the nowhere density and hence the $\sigma-$lower porosity implies the meagerness. Thus if X is a complete space, then $\sigma-$ porous sets are small subsets of X.

It has been done a lot of works connected to the L^p-spaces so far. See for example the first authors works such as [1], [2], [3], [4], [5] and also the work due to Hao., Guo, LI Rong-lu and Wu Jun-de [9], that for Banach spaces X and Y, it is characterized matrix transformations of $\ell^q(X)$ to $\ell^p(Y)$. Furthermore, most of the researches in the topics related to the Lebesgue spaces, have been generalized to the weighted case; see Kuznetsova's works such as [11], that is relevant to the present subject.

Recently, we investigated the existence of $f * g$ as a function for every two function $f, g \in L^p(G, \omega)$, where ω is a submultiplicative weight function on G, and obtained some necessary or sufficient conditions for that the property holds [3]. Mainly, we proved that the $\sigma-$compactness of G is a necessary condition for the existence of $f * g$, when f, g run into $L^p(G,\omega)$. Our purpose of the present work is to consider the Banach space $L^p(G,\omega) \times L^p(G,\omega)$, for $2 < p < \infty$, under the norm

$$\|(f,g)\| = \max\{\|f\|_{p,\omega}, \|g\|_{p,\omega}\}.$$

We then mix some ideas and techniques from [8] and also our paper [3] to prove the following result, as a generalization of Theorems [3, Theorem 2.2] and also [8, Theorem 1.1] whenever $p = q$. In our debate, we use the notation $L^p_\omega \times L^p_\omega$ rather than $L^p(G,\omega) \times L^p(G,\omega)$, in convenience. Let us recall a symmetric weight function ω as $\omega(x) = \widetilde{\omega}(x) = \omega(x^{-1})$, for all $x \in G$. Every symmetric and submultiplicative weight function ω is bounded below by the constant 1, obviously. We state here the main theorem of the present work.

Theorem 1.1. *Let G be a locally compact non $\sigma-$compact group, $2 < p < \infty$ and ω be a symmetric and submultiplicative weight function on G. Then for every compact subset $K \subseteq G$, the set*

$$E^\omega_K = \{(f,g) \in L^p_\omega \times L^p_\omega : \exists x \in K \ \ |f| * |g|(x) < \infty\}$$

is $\sigma - c-$lower porous for some $c > 0$.

2. THE PROOF OF THEOREM 1.1

The proof can be done by slightly modified techniques and methods used in [8, Theorem 1.1], [5, Theorem 2.4] and also [1, Theorem 1.1], as follows. It is easy to see that we can assume K is a compact symmetric neighborhood of the identity element of G with $\lambda(K) > 0$. Since $E^\omega_K = \cup_{n=1}^\infty E^\omega_n$, where

$$E^\omega_n = \{(f,g) \in L^p_\omega \times L^p_\omega : \exists x \in K, \ \ |f| * |g|(x) < n\},$$

thus we only have to show that for each $n \in \mathbb{N}$, the set E^ω_n is $c-$lower porous for some $c > 0$. Suppose that $n \in \mathbb{N}$ and $(f,g) \in E^\omega_n$ and let

$$S = \sup_{x \in K} \Delta(x). \tag{2.1}$$

Submultiplicativity of ω implies that it is bounded and bounded away from zero on every compact subset of G [6, Proposition 1.16]. It follows that there is the number $M \geq 1$ such that for each $x \in K$, $\omega(x) \leq M$. Since $G = \bigcup_{m=1}^\infty \omega^{-1}([1,m])$ together with the fact that G is not $\sigma-$compact, thus there is $m \in \mathbb{N}$ such that $\omega^{-1}([1,m])$ does not contain in any compact subsets of G. Take $d \in (0,1)$ with

$$\left(\frac{d}{1-d}\right)^p + \left(\frac{d}{1-d}\right)^p S \frac{\lambda(K^2)}{\lambda(K)} = 1$$

and let

$$c = \frac{d}{mM^2}.$$

For each $0 < \delta < c$ we have $0 < \delta m M^2 < d < 1$. It follows that

$$\left(\frac{\delta m M^2}{1-\delta m M^2}\right)^p + \left(\frac{\delta m M^2}{1-\delta m M^2}\right)^p S \frac{\lambda(K^2)}{\lambda(K)} < 1.$$

Continuity of the function θ defined by

$$\theta(x) = (\frac{\delta m M^2}{x})^p \left(1 + S\frac{\lambda(K^2)}{\lambda(K)}\right)$$

on $(0,1)$, together with the fact that $\theta(1-\delta m M^2) < 1$ yield that there is $0 < t < 1 - \delta m M^2$ such that $\theta(t) < 1$. Choosing $t < \eta < 1 - \delta$ and $D \in (0,1)$ such that $t = \eta(1-D)$ we obtain

$$P = 1 - \theta(\eta(1-D)) = 1 - \left(\frac{\delta m M^2}{\eta(1-D)}\right)^p - \left(\frac{\delta m M^2}{\eta(1-D)}\right)^p S \frac{\lambda(K^2)}{\lambda(K)} > 0. \tag{2.2}$$

We repeat the argument given in [1] to obtain the sequence (a_k) of $\omega^{-1}([1,m])$ such that for all $l,k \in \mathbb{N}$ with $l \neq k$

$$a_l K^2 \cap a_k K^2 = \emptyset \quad , \quad K a_l^{-1} \cap K a_k^{-1} = \emptyset \quad \textit{and} \quad \Delta(a_k) \leq 1. \tag{2.3}$$

Indeed, take $a_1 \in \omega^{-1}([1,m])$ with $\Delta(a_1) \leq 1$, by the symmetricity of ω. Assume that we have already defined $a_1, \cdots, a_k$. Since ω is symmetric and $\omega^{-1}([1,m])$ is not contained in any compact subsets of G, so we can take

$$a_{k+1} \in \omega^{-1}([1,m]) \setminus \bigcup_{l=1}^{k} (a_l K^4 \cup K^4 a_l^{-1})$$

such that $\Delta(a_{k+1}) \leq 1$. Not that for each $x \in \bigcup_{k=1}^{\infty} a_k K^2 \cup K a_k^{-1}$ we have

$$1 \leq \omega(x) \leq m M^2. \tag{2.4}$$

Given $R > 0$ and set $Q = \dfrac{R}{mM^2}$, since $f, g \in L^p(G,\omega)$, there is $n_0 \in \mathbb{N}$ such that

$$\left(\int_{\cup_{k=n_0}^{\infty} K a_k^{-1}} |f(x)|^p \omega(x)^p \right)^{1/p} \leq (1-\delta-\eta)R \tag{2.5}$$

and

$$\left(\int_{\cup_{k=n_0}^{\infty} a_k K^2} |g(x)|^p \omega(x)^p \right)^{1/p} \leq (1-\delta-\eta)R. \tag{2.6}$$

Choose $n_1 > n_0$ such that

$$(\lambda(K)(n_1 - n_0 + 1))^{1-2/p} > n \left(D^2 \eta^2 Q^2 S^{1/p-1} \left(\frac{\lambda(K)}{\lambda(K^2)} \right)^{1/p} P \right)^{-1} \tag{2.7}$$

and let $A = \cup_{k=n_0}^{n_1} K a_k^{-1}$ and $B = \cup_{k=n_0}^{n_1} a_k K^2$. Then

$$\lambda(A^{-1}) = (n_1 - n_0 + 1)\lambda(K) \quad \textit{and} \quad \lambda(B) = (n_1 - n_0 + 1)\lambda(K^2) \tag{2.8}$$

and so

$$\frac{\lambda(B)}{\lambda(A^{-1})} = \frac{\lambda(K^2)}{\lambda(K)}. \tag{2.9}$$

So by (2.7) and (2.8) we obtain

$$\lambda(A^{-1})^{1-2/p} > n \left(D^2 \eta^2 Q^2 S^{1/p-1} \left(\frac{\lambda(K)}{\lambda(K^2)} \right)^{1/p} P \right)^{-1}. \tag{2.10}$$

Take the positive numbers M_1 and M_2 such that

$$M_1 \lambda(A^{-1})^{1/p} = \eta Q \quad \textit{and} \quad M_2 \lambda(B)^{1/p} = \eta Q. \tag{2.11}$$

Next we define the functions $\widetilde{f}$ and $\widetilde{g}$ the same functions in [8, Theorem 1.1]; i.e. for each $x \in A$, set $\widetilde{f}(x) = M_1 \Delta(x^{-1})^{1/p}$ and $\widetilde{f}(x) = f(x)$ otherwise. Also for each $x \in B$, set $\widetilde{g}(x) = M_2$ and $\widetilde{g}(x) = g(x)$ otherwise. We use (2.4), (2.5) and (2.11) to obtain

$$\|f - \widetilde{f}\|_{p,\omega} \leq \eta R + (1-\delta-\eta)R = (1-\delta)R.$$

Similarly by (2.4), (2.6) and (2.11) we have

$$\|g-\widetilde{g}\|_{p,\omega}\leq(1-\delta)R.$$

It follows that

$$B((\widetilde{f},\widetilde{g}),\delta R)\subseteq B((f,g),R).$$

We should prove that $B((\widetilde{f},\widetilde{g}),\delta R)\cap E_n^{\omega}=\emptyset$. To that end, take $(h,s)\in B((\widetilde{f},\widetilde{g}),\delta R)$ and let

$$A_1=\{x\in A:|h(x)|\geq DM_1\Delta(x^{-1})^{1/p}\}\quad and\quad A_2=A\setminus A_1.$$

Also

$$B_1=\{x\in B:|s(x)|\geq DM_2\}\quad and\quad B_2=B\setminus B_1.$$

Then by (2.4) we have

$$\begin{aligned}\delta R\geq\|h-\widetilde{f}\|_{p,\omega}&\geq\left(\int_{A_2}|h(x)-M_1\Delta(x^{-1})^{1/p}|^p\omega(x)^p d\lambda(x)\right)^{1/p}\\&\geq\left(\int_{A_2}|M_1\Delta(x^{-1})^{1/p}(1-D)|^p\omega(x)^p d\lambda(x)\right)^{1/p}\\&\geq M_1(1-D)\lambda(A_2^{-1})^{1/p}\end{aligned}$$

and (2.11) implies that

$$\lambda(A_2^{-1})\leq\lambda(A^{-1})\left(\frac{\delta mM^2}{\eta(1-D)}\right)^p. \tag{2.12}$$

Similarly

$$\lambda(B_2)\leq\lambda(A^{-1})\frac{\lambda(K^2)}{\lambda(K)}\left(\frac{\delta mM^2}{\eta(1-D)}\right)^p. \tag{2.13}$$

Now let $y_0\in K$ and put $F=(A_1^{-1}y_0)\cap B_1$ and $H=y_0F^{-1}$. Clearly $A^{-1}y_0\subseteq B$, and thus $A_1^{-1}y_0\subseteq B$. The implications (2.12) and (2.13) yield that

$$\begin{aligned}\lambda(H^{-1})&=\lambda(Fy_0^{-1})=\lambda(A_1^{-1})-\lambda(A_1^{-1}\setminus(B_1y_0^{-1}))\\&\geq\lambda(A_1^{-1})-\lambda((B\setminus B_1)y_0^{-1})=\lambda(A_1^{-1})-\lambda(B_2y_0^{-1})\\&=\lambda(A^{-1})-\lambda(A_2^{-1})-\Delta(y_0^{-1})\lambda(B_2)\\&\geq\lambda(A^{-1})P,\end{aligned}$$

where the last inequality is obtained by (2.2), (2.12) and (2.13). Also $H\subseteq A_1$, $F\subseteq B_1$ and $H^{-1}y_0=F$. Furthermore, if $y\in A$ then $y=xa_n^{-1}$ for some $x\in K$ and $n\in\mathbb{N}$ with $n_0\leq n\leq n_1$. Thus by (2.1) and (2.3),

$$\Delta(y^{-1})=\Delta(a_n)\Delta(x^{-1})\leq S.$$

Finally we obtain by (2.8), (2.9), (2.10) and (2.11)

$$\begin{aligned}
\int_H |h(y)||s(y^{-1}y_0)|d\lambda(y) &\geq D^2M_1M_2\int_H \Delta(y^{-1})^{1/p}d\lambda(y)\\
&= D^2M_1M_2\int_H \Delta(y^{-1})^{1/p-1}\Delta(y^{-1})d\lambda(y)\\
&\geq D^2M_1M_2\int_H S^{1/p-1}\Delta(y^{-1})d\lambda(y)\\
&= D^2M_1M_2S^{1/p-1}\int_H \Delta(y^{-1})d\lambda(y)\\
&= D^2M_1M_2S^{1/p-1}\lambda(H^{-1})\\
&\geq D^2M_1M_2S^{1/p-1}\lambda(A^{-1})P\\
&= D^2\eta^2Q^2\left(\frac{\lambda(K)}{\lambda(K^2)}\right)^{1/p}S^{1/p-1}\lambda(A^{-1})^{1-2/p}P\\
&> n.
\end{aligned}$$

It follows that $|h| * |s|(y_0) > n$, for each $y_0 \in K$ and so $(h, s) \notin E_n^{\omega}$, as claimed. □

Remarks 2.1. Let G be a locally compact group, ω be a submultiplicative weight function on G and $2 < p < \infty$.

(i) As we pointed out in the explanations before the theorem, if ω is symmetric and submultiplicative then $\omega(x) \geq 1$, for each $x \in G$. Thus $L^p(G,\omega) \subseteq L^p(G)$. It follows that Theorem 1.1 in the present paper is equivalent to [8, Corollary 2] and with some modifications in the proofs, each of them can be obtained independently from the other one.

(ii) The assumption of non $\sigma-$compactness of G can not be replaced by the non compactness of G, whenever ω is not necessarily a constant function. Indeed, if $f * g$ exists as a function for all $f, g \in L^p(G,\omega)$, then G is $\sigma-$compact and not necessarily compact. Namely take $G = \mathbb{Z}$, the additive group of integer numbers and define the weight function ω on $\mathbb{Z}$ as the following

$$\omega(n) = (1 + |n|) \qquad (n \in \mathbb{Z}).$$

Since $1/\omega \in \ell^q(\mathbb{Z})$, it follows that $f * g$ exists as a function for all $f, g \in \ell^p(\mathbb{Z},\omega)$ [3, Proposition 2.7]. Whereas G is a $\sigma-$compact non-compact group.

Since $L^p(G,\omega) \times L^p(G,\omega)$ is a Banach space, the main results given in [3] are immediately obtained, with observing the proof of Theorem 1.1.

Corollary 2.2. *[3, Theorem 2.2] Let G be a locally compact group, ω be a symmetric and submultiplicative weight function on G and $2 < p < \infty$. If $f * g$ exists for all $f, g \in L^p(G,\omega)$, then $\omega^{-1}(F)$ is contained in a compact subset of G, for all compact subsets F of* $[1,\infty)$,

Corollary 2.3. *[3, Corollary 2.3] Let G be a locally compact group, ω be a symmetric and submultiplicative weight function on G and $2 < p < \infty$. If $f * g$ exists for all $f, g \in L^p(G,\omega)$, then $\omega^{-1}([1,m])$ is contained in a compact subset of G, for all $m \in \mathbb{N}$.*

Recall from [3] that for a submultiplicative weight function ω, the weight function $\omega^* = \omega\widetilde{\omega}$ is a symmetric and submultiplicative weight function on G. Now by using [3, Lemma 2.1] together with Corollary 2.3, the following result is provided.

Corollary 2.4. *[3, Theorem 2.5] Let G be a locally compact group, ω be a submultiplicative weight function on G and $2 < p < \infty$. If $f * g$ exists for all $f, g \in L^p(G,\omega)$, then G is $\sigma-$compact.*

Note 2.5. Note that in the proof of [3, Theorem 2.5], it was used [3, Corollary 2.3] that the continuity of ω is assumed. But the theorem has been based for an arbitrary submultiplicative weight function. The reason is that every submultiplicative weight function ω is equivalent to a continuous weight function α [7, Theorem 2.7]; i.e. there are the positive constants C_1 and C_2 such that

$$C_1 \leq \frac{\omega(x)}{\alpha(x)} \leq C_2,$$

locally almost every where on G. It follows that $L^p(G,\omega) = L^p(G,\alpha)$ and so the result is concluded by replacing α instead of ω; i.e.

$$G = \bigcup_{m=1}^{\infty} (\alpha^*)^{-1}([1,m]).$$

Although, one can get the result without considering this point. Indeed, by [3, Theorem 2.2] for every $m \in \mathbb{N}$, $(\omega^*)^{-1}([1,m])$ is contained in a compact subset of G and since $G = \bigcup_{m=1}^{\infty}(\omega^*)^{-1}([1,m])$ it follows that G is $\sigma-$compact.

Remarks 2.6. Let G be a locally compact group, ω be a weight function on G and $0 < p < \infty$. Then

1. For $0 < p < 1$, with the arguments given in [15], we showed that $f * g$ exists as a function for all $f, g \in L^p(G)$ if and only if G is discrete [2]. Also recently we considered the complete metric space $L^p(G,\omega)$ and showed that if ω is of moderate growth then $L^p(G,\omega)$ is closed under convolution if and only if G is discrete and ω is quasi submultiplicative, that is with some constant $C > 0$,

$$\omega(xy) \leq C\omega(x)\omega(y),$$

   for all $x, y \in G$. Moreover, in the class of submultiplicative weight functions, we proved that $f * g$ exists as a function for all $f, g \in L^p(G,\omega)$ if and only if G is discrete [3].
2. As a known result, $f * g$ exits as a function where $f, g \in L^1(G)$. In the weighted case, $L^1(G,\omega)$ is closed under convolution just when ω is equivalent to a submultiplicative weight function [11, Theorem 3.1]. It follows that for such a weight function, $f * g$ exists as a function for all $f, g \in L^1(G,\omega)$. Note that submultiplicativity of ω is not a necessary condition for the existence of convolution of every two functions belonging to $L^1(G,\omega)$. For example for every bounded below weight function ω, $f * g$ exists as a function for all $f, g \in L^1(G,\omega)$.
3. For $2 < p < \infty$, $f * g$ exists as a function for all $f, g \in L^p(G)$ if and only if G is compact. This subject first was considered by Richert [12]. In fact it was shown that if G is not compact, then for every compact, symmetric neighborhood K of the identity element of G, there exist functions $f, g \in L^p(G)$ such that $f * g(x) = \infty$, for each $x \in K$; see also [1].
4. If ω is submultiplicative, then the existence of $f * g$ as a function for all $f, g \in L^p(G,\omega)$, where $2 < p < \infty$, implies that G is $\sigma-$compact [3, Theorem 2.5]. Note that $\sigma-$compactness of G is not in general a necessary condition whenever ω is not submultiplicative or $1 < p \leq 2$; see [3, Remark 2.6] and also [3, Proposition 2.7].

We end this work with the following example which describes Remark 2.6 for discrete groups.

Examples 2.7. Let G be a discrete group, ω be a submultiplicative weight function on G and $0 < p < \infty$. Then

(1) For $0 < p \leq 2$, $f * g$ exists as a function for all $f, g \in \ell^p(G,\omega)$, by [4, Theorem 2.5], [11, Theorem 3.1] and [3, Remark 2.6,(a)].

(2) If $2 < p < \infty$, then [3, Proposition 2.9] implies that $f * g$ as a function for all $f, g \in \ell^p(G,\omega)$ if and only if $\ell^p(G,\omega)\ \ell^p(G,\widetilde{\omega}) \subseteq \ell^1(G)$, where $\widetilde{\omega}(x) = \omega(x^{-1})$, for all $x \in G$. (Note that there is a misprint in the reference [3, Proposition 2.9] and $\ell^2(G)$ has been printed in stead of $\ell^1(G)$).

References

[1] Abtahi, F., Nasr Isfahani, R. and Rejali, A., *On the L^p-conjecture for locally compact groups*, Arch. Math. (Basel)., **89**, (2007), 237-242.
[2] Abtahi, F., Nasr Isfahani, R. and Rejali, A., *Convolution on L^p-spaces of a locally compact group*, Math. slovaca., **63**, (2013), no 2, 291-298.
[3] Abtahi, F., Nasr Isfahani, R. and Rejali, A., *Convolution on weighted L^p-spaces of locally compact groups*, Proceedings of The Romanian Academy, Series A., **13/2**, (2012), 97-102.
[4] Abtahi, F., *Weighted L^p-spaces on locally compact groups*, Bull. Belg. Math. Soc. Simon Stevin., **19/2**, (2012), 339-343.
[5] Akbarbaglu, I. and Maghsoudi, S., *An answer to a question on the convolution of functions*, Arch. Math. **98**, (2012), 545-553.
[6] Edwards, R. E., *The stability of weighted Lebesgue spaces*, Trans. Amer. Math. Soc., **93**, (1959), 369-394.
[7] Feichtinger, H. G., *Gewichtsfunktionen auf, lokalkompakten Gruppen*, Sitzber. Österr. Akad. Wiss. Abt. II, **188/8-10**, (1979), 451-471.
[8] Glab, S. and Strobin, F., *Porosity and the L^p-conjecture*, Arch. Math., **95**, (2010), 583592.
[9] Hao., Guo, LI Rong-lu and Wu Jun-de., *Matrix transformations of $\ell^q(X)$ to $\ell^p(Y)$*, Appl. Math. J. Chinese Univ., **27/1**, (2012), 78-86.
[10] Hewitt, E. and Ross, K. A., Abstract Harmonic analysis, 2nd edn. **I**, Springer-Verlag, New York, (1970).
[11] Kuznetsova, Yu. N., *Invariant weighted algebras $L^p(G, \omega)$*, Mat. Zametki., **84/4**, (2008), 567-576.
[12] Rickert, N. W., *Convolution of L^p-functions*, Proc. Amer. Math. Soc., **18**, (1967), 762763.
[13] Saeki, S., *The L^p-conjecture and Young's inequality*, Illinois J. Math., **34**, (1990), 615-627.
[14] Zajiček, L., *On $\sigma-$-porous sets in abstract spaces*, Abstr. Appl. Anal., **5**, (2005), 509-534.
[15] Zelasko, W., *On the algebras L^p- of locally compact groups*, Colloq. Math., *8/1* (1961), 112-120.