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On common fixed point and coincidence point theorems in weak partial metric spaces using an implicit relation

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Abstract. In this article, we prove a unique common fixed point and a coincidence point theorems using an implicit relation on weak partial metric spaces. The results presented in this paper extend and generalize several results from the existing literature.

1. Introduction

The study of common fixed points was initiated by Jungck [11] in 1986, and this notion has attracted many researchers to establish the existence of common fixed points by using various contractive conditions.

In 1994, Matthews [13] introduced the concept of partial metric space as a part of the study of denotational semantics of dataflow networks [12, 13, 21, 23]. In partial metric spaces the distance of a point in the self may not be zero. Introducing partial metric space, Matthews extended the Banach contraction principle [4] and proved the fixed point theorem in this space.

Later on Heckmann [9] introduced the concept of weak partial metric space in 1999, which is a generalization of metric space. Some results are recently obtained in [3], [7], [8].

Many classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [14], [15] and in some other papers.

This direction of research produced a consistent literature on fixed point, common fixed point and coincidence point theorems in various ambient spaces. For more details see [1, 5, 6, 10, 16, 17].

In 2013, Vetro and Vetro [22] initiated the study of fixed points of self mappings in partial metric spaces satisfying an implicit relation. In [2], Altun and Turkoglu launched a new type of implicit relation satisfying ϕ-map.

Recently, Popa and Patriciu [18] have studied a new type of ϕ-implicit relation and established a unique point of coincidence and unique common fixed point results and also as application of results they obtained fixed point theorem for a sequence of mappings in partial metric spaces.

More recently, Popa and Patriciu [19] have proved fixed point theorem of Ćirić type in weak partial metric spaces using implicit relation.

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The purpose of this paper is to prove a unique common fixed point and a coincidence point theorems for two self mappings satisfying an implicit relation in the framework of weak partial metric spaces. The results presented in this article extend and generalize several results in the literature.

2. Preliminaries

Now, we give some basic properties and auxiliary results on partial metric space (PMS).

Definition 2.1. *([12, 13]) Let X be a nonempty set and $p\colon X \times X \to [0,\infty)$ be such that for all $x, y, z \in X$ the following postulates are satisfied:*
$(P1)$ $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
$(P2)$ $p(x,x) \le p(x,y)$,
$(P3)$ $p(x,y) = p(y,x)$,
$(P4)$ $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.
Then p is called partial metric on X and the pair (X,p) is called partial metric space.

Remark 2.2. *It is clear that if $p(x,y) = 0$, then $x = y$. But, on the contrary $p(x,x)$ need not be zero.*

Each partial metric space on a set X generates a T_0 topology τ_p on X which has a base the family of open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) \le p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

A sequence $\{x_n\}$ in the partial metric space (X,p) converges with respect to τ_p to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x_n,x)$.

If p is a partial metric on X, then

$$d_w(x,y) = p(x,y) - \min\{p(x,x), p(y,y)\}$$

is an ordinary metric on X.

Remark 2.3. *Let $\{x_n\}$ be a sequence in partial metric space (X,p) and $x \in X$. Then $\lim_{n\to\infty} d_w(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m).$$

Definition 2.4. *([13]) Let (X,p) be a partial metric space. Then*

(1) *a sequence $\{x_n\}$ is called a Cauchy sequence if $\lim_{m,n\to\infty} p(x_m,x_n)$ exists and is finite,*

(2) *(X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that $p(x,x) = \lim_{m,n\to\infty} p(x_m,x_n)$.*

Definition 2.5. *([9]) A weak partial metric space on a nonempty set X is a function $p\colon X \times X \to [0,\infty)$ such that for all $x, y, z \in X$, the followings are satisfied:*
$(WP1)$: $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
$(WP2)$: $p(x,y) = p(y,x)$,
$(WP3)$: $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.
Then p is called weak partial metric on X and the pair (X,p) is called weak partial metric space.

If $p(x,y) = 0$, then $x = y$.

It is obvious that, every partial metric space is a weak partial metric space, but the converse is not true. For example, if $X = [0,\infty)$ and $p(x,y) = \frac{x+y}{2}$, then (X,p) is a weak partial metric space and (X,p) is not a partial metric space. For another example, for $x, y \in \mathbb{R}$ the function $p(x,y) = \frac{e^x+e^y}{2}$ is a non partial metric but weak partial metric on $\mathbb{R}$.

Definition 2.6. *A point x in X is called a coincidence point of two self mappings f and $\mathcal{S}$ of X if $fx = \mathcal{S}x$ for each $x \in X$.*

Theorem 2.7. *([3])) Let (X, p) be a weak partial metric space. Then $d_w \colon X \times X \to [0, \infty)$ is a metric on X.*

Remark 2.8. *In a weak partial metric space, the convergent Cauchy sequence and the completeness are defined as in partial metric space.*

Theorem 2.9. *([3])) Let (X, p) be a weak partial metric space.*
(a) *$\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in (X, d_w).*
(b) *(X, p) is complete if and only if (X, d_w) is complete.*

Lemma 2.10. *([19]) Let (X, p) be a weak partial metric space and $\{x_n\}$ is a sequence in (X, p). If $\lim_{n\to\infty} x_n = x$ and $p(x, x) = 0$, then $\lim_{n\to\infty} p(x_n, y) = p(x, y)$, for all $y \in X$.*

Remark 2.11. *Remark 2.3 is still true for weak partial metric spaces.*

3. Implicit relation

Now, an implicit relation has been introduced to investigate a unique common fixed point, and a coincidence point theorems in weak partial metric spaces.

Definition 3.1. *Let $\mathcal{F}_4$ be the set of all continuous functions $F(t_1, \ldots, t_4) \colon \mathbb{R}_+^4 \to \mathbb{R}$ such that:*
(F_1) : *F is nonincreasing in variable t_4,*
(F_2) : *For all $u, v \geq 0$, there exists $k \in [0, 1)$ such that $F(u, v, \frac{u+v}{4}, \frac{u+v}{4}) \leq 0$ implies $u \leq k\,v$,*
(F_3) : *$F(t, t, \frac{t}{2}, 0) > 0$, for all $t > 0$.*

Example 3.2. $F(t_1, \ldots, t_4) = t_1 - h\ \max\{t_2, 2t_3, 2t_4\}$, *where $k \in [0, 1)$ and $0 \leq k = h < 1$.*

Example 3.3. $F(t_1, \ldots, t_4) = t_1 - h\ \max\{t_2, t_3, t_4\}$, *where $k \in [0, \frac{1}{2})$ and $0 \leq k = h < \frac{1}{2}$.*

Example 3.4. $F(t_1, \ldots, t_4) = t_1 - at_2 - 2bt_3 - 2ct_4$, *where $a, b, c \geq 0$ and $0 < a + b + c < 1$ with $0 \leq k = a + b + c < 1$.*

Example 3.5. $F(t_1, \ldots, t_4) = t_1 - a\ \max\{t_2, t_3\} - 2bt_4$, *where $a, b \geq 0$, $a + b < 1$, with and $0 \leq k = a + b < 1$.*

The purpose of this paper is to establish a unique common fixed point and a coincidence point results under implicit relation in the framework of weak partial metric spaces. The results of findings extend and generalize several results from the existing literature.

4. Main Results

In this section, we shall prove a unique common fixed point and a coincidence point theorems under implicit relation in the framework of weak partial metric spaces.

Theorem 4.1. *Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two self-maps on a complete weak partial metric space (X, p) satisfying the condition:*

$$F\Big(p(\mathcal{T}_1x, \mathcal{T}_2y), p(x, y), \frac{1}{4}\Big[p(x, \mathcal{T}_2y) + p(y, \mathcal{T}_1x)\Big], \frac{1}{4}\Big[p(x, \mathcal{T}_1x) + p(y, \mathcal{T}_2y)\Big]\Big) \leq 0, \tag{1}$$

for all $x, y \in X$, where $F \in \mathcal{F}_4$. Then $\mathcal{T}_1$ and $\mathcal{T}_2$ have a unique common fixed point u in X with $p(u, u) = 0$.

Proof. For each $x_0 \in X$. Let $x_{2n+1} = \mathcal{T}_1 x_{2n} = z_{2n}$ and $x_{2n+2} = \mathcal{T}_2 x_{2n+1} = z_{2n+1}$ for $n = 0, 1, 2, \ldots$. We prove that $\{z_n\}$ is a Cauchy sequence in (X, p). It follows from (1) for $x = z_{2n}$ and $y = z_{2n+1}$ that

$$F\Big(p(\mathcal{T}_1 x_{2n}, \mathcal{T}_2 x_{2n+1}), p(x_{2n}, x_{2n+1}), \frac{1}{4}\Big[p(x_{2n}, \mathcal{T}_2 x_{2n+1}) + p(x_{2n+1}, \mathcal{T}_1 x_{2n})\Big], \frac{1}{4}\Big[p(x_{2n}, \mathcal{T}_1 x_{2n}) + p(x_{2n+1}, \mathcal{T}_2 x_{2n+1})\Big]\Big) \leq 0.$$

$$F\Big(p(z_{2n}, z_{2n+1}), p(z_{2n-1}, z_{2n}), \frac{1}{4}\Big[p(z_{2n-1}, z_{2n+1}) + p(z_{2n}, z_{2n})\Big], \frac{1}{4}\Big[p(z_{2n-1}, z_{2n}) + p(z_{2n}, z_{2n+1})\Big]\Big) \leq 0. \tag{2}$$

Since by $(WP3)$,

$$p(z_{2n-1}, z_{2n+1}) \leq p(z_{2n-1}, z_{2n}) + p(z_{2n}, z_{2n+1}) - p(z_{2n}, z_{2n}). \tag{3}$$

By (2), (3) and (F_1), we obtain

$$F\Big(p(z_{2n}, z_{2n+1}), p(z_{2n-1}, z_{2n}), \frac{1}{4}\Big[p(z_{2n-1}, z_{2n}) + p(z_{2n}, z_{2n+1})\Big], \frac{1}{4}\Big[p(z_{2n-1}, z_{2n}) + p(z_{2n}, z_{2n+1})\Big]\Big) \leq 0. \tag{4}$$

By (F_2) there exists $k \in [0, 1)$, we obtain

$$p(z_{2n}, z_{2n+1}) \leq k\, p(z_{2n-1}, z_{2n}).$$

By similar process using equation (1) for $x = x_{2n+2}$ and $y = x_{2n+1}$, we obtain

$$p(z_{2n+2}, z_{2n+1}) \leq k\, p(z_{2n+1}, z_{2n}),$$

which implies

$$p(z_n, z_{n+1}) \leq k\, p(z_{n-1}, z_n) \leq k^2\, p(z_{n-2}, z_{n-1}) \leq \cdots \leq k^n\, p(z_0, z_1).$$

For $n, m \in \mathbb{N}$ with $m > n$, by repeated use of $(WP3)$, we have that

$$\begin{aligned}
p(z_n, z_m) &\leq p(z_n, z_{n+1}) + p(z_{n+1}, z_{n+2}) + \cdots + p(z_{m-1}, z_m) \\
&\quad - p(z_{n+1}, z_{n+1}) - p(z_{n+2}, z_{n+2}) - \cdots - p(z_{m-1}, z_{m-1}) \\
&\leq p(z_n, z_{n+1}) + p(z_{n+1}, z_{n+2}) + \cdots + p(z_{m-1}, z_m) \\
&\leq (k^n + k^{n+1} + \cdots + k^m) p(z_0, z_1) \\
&\leq \Big(\frac{k^n}{1-k}\Big) p(z_0, z_1) \to 0 \text{ as } n \to \infty.
\end{aligned}$$

By the definition of $d_w(x, y)$ we obtain

$$d_w(z_n, z_m) \leq p(z_n, z_m) \to 0 \text{ as } n, m \to \infty.$$

This implies that $\{z_n\}$ is a Cauchy sequence in (X, d_w). By Theorem 2.9, $\{z_n\}$ is a Cauchy sequence in (X, p). Since (X, p) is complete, $\{z_n\}$ converges in (X, p) to a point $u \in X$ and $u = \lim_{n\to\infty} z_n$. By Theorem 2.9, we obtain

$$p(u, u) = \lim_{n\to\infty} p(u, z_n) = \lim_{n,m\to\infty} p(z_n, z_m) = 0. \tag{5}$$

Now, we show that u is a common fixed point of $\mathcal{T}_1$ and $\mathcal{T}_2$. Notice that due to (5), we have $p(u,u) = 0$. By (1) with $x = u$ and $y = x_{2n+1}$ and using (5), we have

$$F\Big(p(\mathcal{T}_1 u, \mathcal{T}_2 x_{2n+1}), p(u, x_{2n+1}), \frac{1}{4}\big[p(u, \mathcal{T}_2 x_{2n+1}) + p(x_{2n+1}, \mathcal{T}_1 u)\big], \frac{1}{4}\big[p(u, \mathcal{T}_1 u) + p(x_{2n+1}, \mathcal{T}_2 x_{2n+1})\big]\Big) \leq 0.$$

$$F\Big(p(\mathcal{T}_1 u, x_{2n+2}), p(u, x_{2n+1}), \frac{1}{4}\big[p(u, x_{2n+2}) + p(x_{2n+1}, \mathcal{T}_1 u)\big], \frac{1}{4}\big[p(u, \mathcal{T}_1 u) + p(x_{2n+1}, x_{2n+2})\big]\Big) \leq 0. \tag{6}$$

Letting $n \to \infty$ in (6) and using $(WP2)$, we obtain by Lemma 2.10 that

$$F\Big(p(\mathcal{T}_1 u, u), 0, \frac{1}{4}\big[p(\mathcal{T}_1 u, u) + 0\big], \frac{1}{4}\big[p(\mathcal{T}_1 u, u) + 0\big]\Big) \leq 0.$$

By (F_2) we obtain $p(\mathcal{T}_1 u, u) = 0$. Hence, $\mathcal{T}_1 u = u$. This shows that u is a fixed point of $\mathcal{T}_1$. Similarly, we can show that $\mathcal{T}_2 u = u$. Thus u is a common fixed point of $\mathcal{T}_1$ and $\mathcal{T}_2$.

Now, we have to show that the common fixed point of $\mathcal{T}_1$ and $\mathcal{T}_2$ is unique. Assume that u' is another common fixed point of $\mathcal{T}_1$ and $\mathcal{T}_2$ such that $\mathcal{T}_1 u' = u' = \mathcal{T}_2 u'$ with $u \neq u'$. Now using (1), (5) and $(WP2)$ with $x = u$ and $y = u'$, we have

$$F\Big(p(\mathcal{T}_1 u, \mathcal{T}_2 u'), p(u, u'), \frac{1}{4}\big[p(u, \mathcal{T}_2 u') + p(u', \mathcal{T}_1 u)\big], \frac{1}{4}\big[p(u, \mathcal{T}_1 u) + p(u', \mathcal{T}_2 u')\big]\Big) \leq 0.$$

$$F\Big(p(u, u'), p(u, u'), \frac{1}{4}\big[p(u, u') + p(u', u)\big], \frac{1}{4}\big[p(u, u) + p(u', u')\big]\Big) \leq 0.$$

$$F\Big(p(u, u'), p(u, u'), \frac{p(u, u')}{2}, 0\Big) \leq 0,$$

a contradiction of (F_3) if $p(u,u') > 0$. Hence, $p(u,u') = 0$ which implies $u = u'$. This shows that the common fixed point of $\mathcal{T}_1$ and $\mathcal{T}_2$ is unique. This completes the proof. □

Theorem 4.2. *Let $\mathcal{S}$ and g be two self-maps on a complete weak partial metric space (X, p) satisfying the condition:*

$$F\Big(p(\mathcal{S}x, \mathcal{S}y), p(gx, gy), \frac{1}{4}\big[p(gx, \mathcal{S}y), p(gy, \mathcal{S}x)\big], \frac{1}{4}\big[p(gx, \mathcal{S}x) + p(gy, \mathcal{S}y)\big]\Big) \leq 0, \tag{7}$$

for all $x, y \in X$, where $F \in \mathcal{F}_4$. If the range of g contains the range of $\mathcal{S}$ and $g(X)$ is a complete subspace of X, then $\mathcal{S}$ and g have a coincidence fixed point with $p(gv, gv) = 0$.

Proof. Let $x_0 \in X$ and choose a point x_1 in X such that $\mathcal{S}x_0 = gx_1, \ldots, \mathcal{S}x_n = gx_{n+1} = w_{n+1}$. Then from (7) for $x = x_{n-1}$ and $y = x_n$, we have successively

$$F\Big(p(\mathcal{S}x_{n-1}, \mathcal{S}x_n), p(gx_{n-1}, gx_n), \frac{1}{4}\big[p(gx_{n-1}, \mathcal{S}x_n) + p(gx_n, \mathcal{S}x_{n-1})\big], \frac{1}{4}\big[p(gx_{n-1}, \mathcal{S}x_{n-1}) + p(gx_n, \mathcal{S}x_n)\big]\Big) \leq 0.$$

$$F\Big(p(w_n, w_{n+1}), p(w_{n-1}, w_n), \frac{1}{4}\big[p(w_{n-1}, w_{n+1}) + p(w_n, w_n)\big], \frac{1}{4}\big[p(w_{n-1}, w_n) + p(w_n, w_{n+1})\big]\Big) \leq 0. \tag{8}$$

Since by $(WP3)$,

$$p(w_{n-1}, w_{n+1}) \leq p(w_{n-1}, w_n) + p(w_n, w_{n+1}) - p(w_n, w_n). \tag{9}$$

By (8), (9) and (F_1), we obtain

$$F\Big(p(w_n, w_{n+1}), p(w_{n-1}, w_n), \frac{1}{4}\big[p(w_{n-1}, w_n) + p(w_n, w_{n+1})\big], \frac{1}{4}\big[p(w_{n-1}, w_n) + p(w_n, w_{n+1})\big]\Big) \leq 0. \tag{10}$$

By (F_2), there exists $k \in [0, 1)$ such that

$$p(w_n, w_{n+1}) \leq k\, p(w_{n-1}, w_n),$$

which implies

$$p(w_n, w_{n+1}) \leq k\, p(w_{n-1}, w_n) \leq k^2\, p(w_{n-2}, w_{n-1}) \leq \cdots \leq k^n\, p(w_0, w_1).$$

For $n, m \in \mathbb{N}$ with $m > n$, by repeated use of $(WP3)$, we have that

$$\begin{aligned} p(w_n, w_m) &\leq p(w_n, w_{n+1}) + p(w_{n+1}, w_{n+2}) + \cdots + p(w_{m-1}, w_m) \\ &\quad - p(w_{n+1}, w_{n+1}) - p(w_{n+2}, w_{n+2}) - \cdots - p(w_{m-1}, w_{m-1}) \\ &\leq p(w_n, w_{n+1}) + p(w_{n+1}, w_{n+2}) + \cdots + p(w_{m-1}, w_m) \\ &\leq (k^n + k^{n+1} + \cdots + k^m) p(w_0, w_1) \\ &\leq \Big(\frac{k^n}{1-k}\Big) p(w_0, w_1) \to 0 \text{ as } n \to \infty. \end{aligned}$$

By the definition of $d_w(x, y)$ we obtain

$$d_w(w_n, w_m) \leq p(w_n, w_m) \to 0 \,\text{as}\, n, m \to \infty.$$

This implies that $\{w_n\}$ is a Cauchy sequence in (X, d_w). By Theorem 2.9, $\{w_n\}$ is a Cauchy sequence in (X, p). Since (X, p) is complete, $\{w_n\}$ converges in (X, p) to a point $v \in X$ such that $w_n \to v \Rightarrow g w_n \to g v$ as $n \to \infty$, since $g(X)$ is a complete subspace of X. Moreover by Lemma 2.10,

$$p(gv, gv) = \lim_{n\to\infty} p(gv, gw_n) = \lim_{n,m\to\infty} p(gw_n, gw_m) = 0, \tag{11}$$

Now, we show that v is a coincidence point of $\mathcal{S}$ and g. Notice that due to (11), we have $p(gv, gv) = 0$. By (7) with $x = v$ and $y = w_n$, we have

$$F\Big(p(\mathcal{S}v, \mathcal{S}w_n), p(gv, gw_n), \frac{1}{4}\big[p(gv, \mathcal{S}w_n) + p(gw_n, \mathcal{S}v)\big], \frac{1}{4}\big[p(gv, \mathcal{S}v) + p(gw_n, \mathcal{S}w_n)\big]\Big) \leq 0.$$

$$F\Big(p(\mathcal{S}v, gw_{n+1}), p(gv, gw_n), \frac{1}{4}\big[p(gv, gw_{n+1}) + p(gw_n, \mathcal{S}v)\big], \frac{1}{4}\big[p(gv, \mathcal{S}v) + p(gw_n, gw_{n+1})\big]\Big) \leq 0. \tag{12}$$

Letting $n \to \infty$ in (12) and using $(WP2)$, we obtain by Lemma 2.10 that

$$F\Big(p(\mathcal{S}v, gv), 0, \frac{1}{4}\big[p(\mathcal{S}v, gv) + 0\big], \frac{1}{4}\big[p(\mathcal{S}v, gv) + 0\big]\Big) \leq 0. \tag{13}$$

By (F_2) we obtain $p(\mathcal{S}v, gv) = 0$. Hence, $\mathcal{S}v = gv$. This shows that v is a coincidence point of $\mathcal{S}$ and g. This completes the proof. □

If take $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}$ in Theorem 4.1, then we have the following result.

Corollary 4.3. *Let $\mathcal{T}$ be a self-map on a complete weak partial metric space (X, p) satisfying the condition:*

$$F\Big(p(\mathcal{T}x, \mathcal{T}y), p(x, y), \frac{1}{4}\Big[p(x, \mathcal{T}y) + p(y, \mathcal{T}x)\Big], \frac{1}{4}\Big[p(x, \mathcal{T}x) + p(y, \mathcal{T}y)\Big]\Big) \leq 0,$$

for all $x, y \in X$, where $F \in \mathcal{F}_4$. Then $\mathcal{T}$ has a unique fixed point z in X with $p(z, z) = 0$.

By Corollary 4.3 and Example 3.2 we obtain a theorem of Rhoades type ([20]) in complete weak partial metric spaces.

Theorem 4.4. *Let $\mathcal{T}$ be a self-map on a complete weak partial metric space (X, p) such that for all $x, y \in X$*

$$p(\mathcal{T}x, \mathcal{T}y) \leq k \max\Big\{p(x, y), \frac{1}{2}\Big[p(x, \mathcal{T}y) + p(y, \mathcal{T}x)\Big], \frac{1}{2}\Big[p(x, \mathcal{T}x) + p(y, \mathcal{T}y)\Big]\Big\}$$

where $k \in [0, 1)$. Then $\mathcal{T}$ has a unique fixed point z in X with $p(z, z) = 0$.

Remark 4.5. *By Corollary 4.3 and Example 3.3-3.5 we obtain new results.*

Remark 4.6. *If we take $g = I$, the identity map and $\mathcal{S} = \mathcal{T}$ is the single valued mapping in Theorem 4.2, then we obtain Corollary 4.3 of this paper.*

5. Conclusion

In this paper, we establish a unique common fixed point and a coincidence point theorems under an implicit relation in the framework of weak partial metric spaces. The results presented in this article extend and generalize several results in the existing literature.

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