Functional Analysis,
Approximation and
Computation
17:2 (2025), 67–72

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/faac

Fixed point theorems of enriched contraction mappings in n-Banach spaces

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Abstract. In this paper, we introduce the (b, θ)-enriched contraction on a nonempty, closed, convex and bounded subset of an n-Banach space. In particular, we discuss the existence and uniqueness of a fixed point of such a mapping in nonempty, closed, convex and bounded subsets of an n-Banach space. Moreover, many results are established in the framework of n-Banach spaces.

1. Introduction and preliminaries

Berinde and Păcurar [3] introduced enriched contraction mappings which include the class of Banach contractive mappings. They proved that an enriched contraction mapping on a Banach space has an unique fixed point, which can be approximated by means of the Krasnoselskii iterative scheme. Recently, Anjum and Abbas [1] demonstrated that an enriched contraction mapping on a nonempty, closed, bounded and convex subset of a 2-Banach space. If $\mathcal{M}$ is a convex subset of the space $\mathcal{E}$, $S : \mathcal{M} \to \mathcal{M}$ is a self-mapping on $\mathcal{M}$ and $\lambda \in (0, 1]$, then the mapping $S_\lambda : \mathcal{M} \to \mathcal{M}$ given by

$$S_\lambda(x) = (1 - \lambda)x + \lambda Sx \qquad \text{for any } x \in \mathcal{M} \tag{1}$$

is called an averaged mapping. A Picard iteration $\{x_n : n = 0, 1, 2, ...\}$ corresponding to an averaged mapping S_λ is called a Krasnoselskii iteration.

It was demonstrated in [3] that all enriched contraction on a Banach space has an unique fixed point, which can be approximated by means of the Krasnoselskii iterative scheme.

In [9], Gähler introduced and studied the notions of 2-normed spaces and 2-metric spaces. Recently, White [17] initiated and studied the 2-Banach spaces. The n-normed spaces and n-Banach spaces were studied by several researchers, see [8, 11–13].

2020 *Mathematics Subject Classification*. Primary 47H10.
Keywords. Fixed point theorems; Enriched contraction mappings; n-Banach spaces; Closed and bounded sets.
Received: 29 September 2025; Accepted: 12 October 2025
Communicated by Dragan S. Djordjević
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Definition 1.1 ([3]). *Let $\mathcal{E}$ be a linear normed space. A mapping $S : \mathcal{E} \to \mathcal{E}$ is said to be a (b, θ)-enriched contraction if there exist $b \in [0, \infty)$ and $\theta \in [0, b+1)$ such that*

$$\left\|b(x-y) + Sx - Sy\right\| \leq \theta \left\|x - y\right\| \qquad \text{for all } x, y \in \mathcal{E}. \tag{2}$$

Note that a $(0, \theta)$-enriched contraction mapping is a Banach contraction.

Theorem 1.2 ([3]). *Let $\mathcal{E}$ be a Banach space and $S : \mathcal{E} \to \mathcal{E}$ a (b, θ)-enriched contraction. Then*
(i) Fix $(S) = \{p\}$;
(ii) There exists $\lambda \in (0, 1]$ such that the iterative method $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1-\lambda)x_n + \lambda S x_n, \; n \geq 0, \tag{3}$$

converges to p, for any $x_0 \in \mathcal{E}$;
(iii) The following estimate holds

$$\|x_{n+i-1} - p\| \leq \frac{c^i}{1-c} \cdot \|x_n - x_{n-1}\|, \quad n = 0, 1, 2, \ldots; \; i = 1, 2, \ldots, \tag{4}$$

where $c = \dfrac{\theta}{b+1}$.

Theorem 1.3 ([3]). *Let $\mathcal{E}$ be a Banach space and let $S : \mathcal{E} \to \mathcal{E}$ be a mapping with the property that there exists a positive integer N such that S^N is a (b, θ)-enriched contraction. Then*
(i) Fix $(S) = \{p\}$;
(ii) There exists $\lambda \in (0, 1]$ such that the iterative method $\{x_n\}_{n=0}^{\infty}$ given by

$$x_{n+1} = (1-\lambda)x_n + \lambda S^N x_n, \; n \geq 0,$$

converges to p, for any $x_0 \in \mathcal{E}$.

Definition 1.4 ([11–13]). *Let $\mathcal{E}$ be a real vector space with $\dim \mathcal{E} \geq n$. A function $\|\cdot, \cdots, \cdot\| : \mathcal{E}^n \to \mathbb{R}^+$ is called an n-norm if*

- *(i) $\|x_1, \cdots, x_n\| = 0$ if and only if $x_1, \cdots, x_n$ are linearly dependent.*
- *(ii) $\|x_1, \cdots, x_n\|$ is invariant under permutation.*
- *(iii) For each $x_1, \cdots, x_n \in \mathcal{E}$ and for all $\lambda \in \mathbb{R}$, $\|\lambda x_1, \cdots, x_n\| = |\lambda| \|x_1, \cdots, x_n\|$.*
- *(iv) For all $x, y, x_2 \cdots, x_n \in \mathcal{E}$, $\|x + y, x_2, \cdots, x_n\| \leq \|x, x_2, \cdots, x_n\| + \|y, x_2, \cdots, x_n\|$.*

Then $(\mathcal{E}, \|\cdot, \cdots, \cdot\|)$ is called an n-normed space.

Hereafter, let $(\mathcal{E}, \|\cdot, \cdots, \cdot\|)$ be an n-normed space and $\mathcal{A} = \{a_1, a_2, \cdots, a_n\}$ be a collection of n-linearly independent vectors in $\mathcal{E}$. We recall some nedeed results:

Definition 1.5 ([14]). *A sequence $(x_k)_{k \in \mathbb{N}} \subset \mathcal{E}$ is said to be Cauchy sequence with respect to $\mathcal{A}$ if $\lim_{m,k \to \infty} \|x_k - x_m, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$. If every Cauchy sequence in $\mathcal{E}$ converges to some $x \in \mathcal{E}$, then $(\mathcal{E}, \|\cdot, \cdots, \cdot\|)$ is said to be complete with respect to $\mathcal{A}$.*

Definition 1.6 ([14]). *Let $B \subset \mathcal{E}$ be a nonempty set. Then we say that B is closed if for every sequence $(x_k)_{k \in \mathbb{N}}$ in B which converges in $\mathcal{E}$, its limit is in B.*

Definition 1.7 ([14]). *A sequence $(x_k)_{k \in \mathbb{N}} \subset \mathcal{E}$ converges to an element $x \in \mathcal{E}$ with respect to $\mathcal{A}$ denoted by $x_k \xrightarrow{\mathcal{A}} x$ as $k \to \infty$ if there exists an element $x \in \mathcal{E}$ such that $\lim_{k \to \infty} \|x_k - x, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$.*

Definition 1.8 ([14]). *Let B be a nonempty subset of $\mathcal{E}$, then B is called bounded with respect to $\mathcal{A}$ if there exists $M > 0$ such that*

$$\|x, a_{i_2}, \cdots, a_{i_n}\| \leq M \tag{5}$$

for any $x \in \mathcal{E}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$.

Lemma 1.9 ([14]). *Let $x \in \mathcal{E}$. If $\|x, a_{i_2}, \cdots, a_{i_n}\| = 0$ for any $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$, then $x = 0$.*

Definition 1.10 ([14]). *Let $\mathcal{E}$ be an n-Banach space. A mapping $S : \mathcal{M} \to \mathcal{M}$ is said to be a contraction mapping with respect to $\mathcal{A}$ if there exists $\lambda \in (0, 1)$ such that*

$$\|Sx - Sx, a_{i_2}, \cdots, a_{i_n}\| \leq \lambda \|x - y, a_{i_2}, \cdots, a_{i_n}\| \tag{6}$$

for all $x, y \in \mathcal{E}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$.

Theorem 1.11 ([14]). *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed, and bounded with respect to $\mathcal{A}$. If $S : \mathcal{M} \longrightarrow \mathcal{M}$ is a contraction mapping with respect to $\mathcal{A}$, then S has a unique fixed point in $\mathcal{M}$.*

Theorem 1.12 ([14]). *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed and bounded with respect to $\mathcal{A}$. If $S : \mathcal{M} \longrightarrow \mathcal{M}$ is a φ-contraction mapping with respect to $\mathcal{A}$, then S has a unique fixed point in $\mathcal{M}$.*

2. Main results

Now, we introduce the (b, θ)-enriched contraction mapping with respect to $\mathcal{A}$ in n-normed spaces.

Definition 2.1. *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed and bounded with respect to $\mathcal{A}$. A mapping $S : \mathcal{M} \to \mathcal{M}$ is said to be a (b, θ)-enriched contraction with respect to $\mathcal{A}$, if there exist $b \in [0, \infty)$ and $\theta \in [0, b + 1)$ such that for each $x, y \in \mathcal{M}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$, we have*

$$\left\|b(x - y) + Sx - Sy, a_{i_2}, \cdots, a_{i_n}\right\| \leq \theta \left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\|. \tag{7}$$

The following example shows that the class of all (b, θ)-enriched contractions is nonempty.

Example 2.2. *(i) Any contraction S defined as in (6) with contractive constant $\lambda \in [0, 1)$ is a $(0, \lambda)$-enriched contraction with respect to $\mathcal{A}$.*

(ii) Let $\mathcal{E}$ be an n-normed space, $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed and bounded with respect to $\mathcal{A}$ and let w be an arbitrary but fixed element of $\mathcal{M}$. Define a mapping $S : \mathcal{M} \to \mathcal{M}$ by

$$Sx = w - x.$$

Then S is not a contraction in the sense of (6). However, let us show that S is a (b, θ)-enriched contraction with respect to $\mathcal{A}$. Indeed, in this case, the enriched contractivity condition (7) is equivalent to

$$|b - 1| \left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\| \leq \theta \left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\| \tag{8}$$

for all $x, y \in \mathcal{M}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$ where $\theta \in [0, b + 1)$. The above inequality is valid for all $x, y \in \mathcal{M}$ if one chooses any $b \in (0, 1)$ and $\theta = 1 - b$. Hence, for any $b \in (0, 1)$, S is a $(b, 1 - b)$-enriched contraction with respect to $\mathcal{A}$. Notice that $w/2$ is a fixed point of S because $S(w/2) = w/2$.

Remark 2.3. *We highlight that the Picard iteration $\{x_n\}$ associated to the mapping S defined by (8) in Example (2.2) does not converge whatever the initial point x_0 distinct from $w/2$.*

The Remark (2.3) illustrates that the Picard iteration is not useful in order to approximate fixed points of enriched contractions. Then we need to change our point of view. In the rest of the section, we are going to demonstrate that the Krasnoselskii iterative scheme [5] is more appropriate in this context. In fact, we shall demonstrate a strong convergence theorem for the class of enriched contractions. Before that, we recall the following notion:

Remark 2.4. *Let $\mathcal{M}$ be a convex subset of an n-normed space $\mathcal{E}$ and let $S : \mathcal{M} \to \mathcal{M}$. Then for each $\lambda \in (0, 1]$, the set of any fixed points of the averaged mapping $S_\lambda : \mathcal{M} \to \mathcal{M}$ given by $S_\lambda(x) = (1 - \lambda)x + \lambda Sx$ for each $x \in \mathcal{M}$ coincides with set of any fixed points of S.*

We now give a generalization of Theorem 1.11 for the case of (b, θ)-enriched contractions in n-Banach spaces.

Theorem 2.5. *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed, bounded and convex subset with respect to $\mathcal{A}$. Let $S : \mathcal{M} \to \mathcal{M}$ be a (b, θ)-enriched contraction. Suppose that there exists $x_0 \in \mathcal{M}$ such that $x_0 - S_\lambda x_0 \in \mathcal{M}$, then*

1. *$Fix(S) = \{x^*\}$;*
2. *The iterative sequence $\{x_n\}_{n=1}^{\infty}$ given by*

$$x_n = (1 - \lambda)x_{n-1} + \lambda Sx_{n-1} \qquad \text{for all } n \in \mathbb{N} \tag{9}$$

converges to x^*, where $\lambda = 1/(b + 1)$.

Proof. From $\lambda = 1/(b + 1)$, we obtain $b = (1 - \lambda)/\lambda$. Let $S_\lambda : \mathcal{M} \to \mathcal{M}$ be given as in (1). Since S is a (b, θ)-enriched contraction with respect to $\mathcal{A}$, we get for all $x, y \in \mathcal{M}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$,

$$\begin{aligned}
\frac{1}{\lambda}\left\|S_\lambda x - S_\lambda y, a_{i_2}, \cdots, a_{i_n}\right\| &= \frac{1}{\lambda}\left\|[\,(1-\lambda)x + \lambda Sx\,] - [\,(1-\lambda)y + \lambda Sy\,], a_{i_2}, \cdots, a_{i_n}\right\| \\
&= \frac{1}{\lambda}\left\|(1-\lambda)(x-y) + \lambda(Sx - Sy), a_{i_2}, \cdots, a_{i_n}\right\| \\
&= \left\|\frac{1-\lambda}{\lambda}(x-y) + (Sx - Sy), a_{i_2}, \cdots, a_{i_n}\right\| \\
&= \left\|b(x-y) + (Sx - Sy), a_{i_2}, \cdots, a_{i_n}\right\| \\
&\le \theta\left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\|.
\end{aligned}$$

So

$$\left\|S_\lambda x - S_\lambda y, a_{i_2}, \cdots, a_{i_n}\right\| \le d\left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\| \tag{10}$$

for each $x, y \in \mathcal{M}$ and $\{i_2, \cdots, i_n\} \subset \{1, 2, \cdots, n\}$, where $d = \theta\lambda$. Since $\theta \in (0, b + 1)$, then $d \in (0, 1)$. Now, by Theorem 1.11, S_λ has a unique fixed point in $\mathcal{M}$ denoted x^*. By Remark 2.4, as the sets of fixed points of S and S_λ coincide, we conclude that $Sx^* = x^*$ and that x^* is the unique fixed point of S. □

Remark 2.6. *If S is a $(0, \theta)$-enriched contraction with respect to $\mathcal{A}$, then S is contraction satisfying for each $x, y \in \mathcal{M}$,*

$$\left\|Sx - Sy, a_{i_2}, \cdots, a_{i_n}\right\| \le \theta\left\|x - y, a_{i_2}, \cdots, a_{i_n}\right\|.$$

In this case, the Krasnoselskii type iterative process (9) *reduces to the Picard iterative process (because $b = 0$ implies $\lambda = 1$ in Theorem 2.5). Hence Theorem 2.5 states that the Picard iterative process can be applied for contractions in an n-metric space.*

If we take $b = 0$ in the Theorem 2.5, we obtain Theorem 1.11 in the setting of closed, bounded and convex subset with respect to $\mathcal{A}$ in an n-Banach space.

Theorem 2.7 ([14]). *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed, bounded and convex subset with respect to $\mathcal{A}$. Let $S : \mathcal{M} \to \mathcal{M}$ be a $(0, \theta)$-enriched contraction with respect to $\mathcal{A}$, then S has unique fixed point.*

3. Local and asymptotic versions of the enriched contractive principle

In concrete applications, the local variant of the enriched contraction (which involves an open ball B in a Banach space X and a nonself enriched contractive map of B into X with an essential property that it does not displace the center of the ball too far) is important (see [3]). The analog of this result in the case of (b, θ)-enriched contractions in 2-Banach spaces is given by the following result.

Corollary 3.1. *Let $\mathcal{E}$ be an n-Banach space, $x_0, e_2, \cdots e_n \in \mathcal{E}$, $r > 0$, $B = B_{e_2,\cdots e_n}(x_0, r)$ and $S : B \to \mathcal{E}$ a (b, θ)-enriched contraction. If*

$$\|x_0 - Sx_0, e_2, \cdots e_n\| < (b + 1 - \theta)r, \tag{11}$$

then S has a unique fixed point provided that B is bounded and $S_\lambda(B) \subseteq B$, where $\lambda = \frac{1}{b+1}$.

Proof. Let $\varepsilon \in (0, r)$ be such that

$$\|x_0 - Sx_0, e_2, \cdots e_n\| < (b + 1 - \theta)\varepsilon < (b + 1 - \theta)r.$$

In a similar manner to the proof of Theorem 2.5, one obtains that S_λ is a $(0, d)$-enriched contraction on B, that is, we have

$$\left\|S_\lambda x - S_\lambda y, z_2, \cdots, z_n\right\| \leq d \left\|x - y, z_2, \cdots, z_n\right\| \qquad \text{for each } x, y, z_2, \cdots, z_n \in B, \tag{12}$$

where

$$d = \theta\lambda = \frac{\theta}{b + 1}.$$

From the inequality (11), one obtains that

$$\|x_0 - S_\lambda x_0, e_2, \cdots e_n\| < \left(1 - \frac{\theta}{b + 1}\right)\varepsilon = (1 - d)\varepsilon. \tag{13}$$

We now prove that the closed ball

$$A = B_{e_2,\cdots e_n}[x_0, \varepsilon] = \{x \in \mathcal{E} : \|x_0 - x, e_2, \cdots e_n\| \leq \varepsilon\} \subseteq B$$

is invariant under S_λ. Indeed, for any $x \in A$, using triangular inequality, (12) and (13), we have

$$\begin{aligned}\|x_0 - S_\lambda x, e_2, \cdots e_n\| &= \|x_0 - S_\lambda x_0 + S_\lambda x_0 - S_\lambda x, e_2, \cdots e_n\| \\ &\leq \|x_0 - S_\lambda x_0, e_2, \cdots e_n\| + \|S_\lambda x_0 - S_\lambda x, e_2, \cdots e_n\| \\ &< (1 - d)\varepsilon + d\,\|x_0 - x, e_2, \cdots e_n\| \leq (1 - d)\varepsilon + d\varepsilon = \varepsilon,\end{aligned}$$

that is $S_\lambda x \in A$, for any $x \in A$. The conclusion follows by Theorem 2.5. □

Now, we present the local version in the setting of a nonempty, closed, convex and bounded subset of an n-Banach space.

Theorem 3.2. *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed, convex and bounded with respect to $\mathcal{A}$. and $S : \mathcal{M} \to \mathcal{E}$ be a (b, θ)-enriched contraction with respect to $\mathcal{A}$. Then S has a unique fixed point provided that $b > 0$.*

Proof. Since $b > 0$, then $\lambda = \frac{1}{b+1} \in (0, 1)$. As $\mathcal{M}$ is convex, $S_\lambda x \in \mathcal{M}$ for all $x \in \mathcal{M}$. As S is a (b, θ)-enriched contraction, one obtains from condition (7) that S_λ is a $(0, d)$-enriched contraction on $\mathcal{M}$, where

$$d = \theta\lambda = \frac{\theta}{b + 1}.$$

Since $\theta \in (0, b+1)$, we obtain $d \in (0,1)$. Let $x_0 \in \mathcal{M}$. Hence we can construct a sequence $\{x_n\}_{n\in\mathbb{N}}$ in a similar manner as in the proof of Theorem 2.5, and one obtains

$$\left\|x_n - x_m, a_{i_2}, \cdots, a_{i_n}\right\| \leq \frac{d^n}{1-d} \cdot \left\|x_0 - x_1, a_{i_2}, \cdots, a_{i_n}\right\|. \tag{14}$$

Hence, it can be proved that $\{x_n\}$ is a Cauchy sequence. Following similar arguments to those given in the proof of Theorem 2.5, the result follows. □

Our next theorem is the more general case of (b, θ)-enriched contractions in n-Banach spaces.

Theorem 3.3. *Let $\mathcal{E}$ be an n-Banach space and $\mathcal{M} \subset \mathcal{E}$ be a nonempty, closed, convex and bounded with respect to $\mathcal{A}$. Let $S : \mathcal{M} \to \mathcal{M}$ be a mapping with the property there exists a positive integer N such that S^N is a (b, θ)-enriched contraction. Assume that there exists $x_0 \in \mathcal{M}$ such that $x_0 - S^N_\lambda x_0 \in \mathcal{M}$. Then*

1. *$Fix(S) = \{x^*\}$;*
2. *the iterative method $\{x_n\}_{n=1}^{\infty}$ given by*

$$x_n = (1-\lambda)x_{n-1} + \lambda S^N x_{n-1} \qquad \textit{for all } n \in \mathbb{N} \tag{15}$$

converges to x^*, where $\lambda = 1/(b+1)$.

Proof. We apply Theorem 2.5 to the mapping S^N and obtain that $Fix(S^N) = \{x^*\}$. We also have

$$S^N(S(x^*)) = S^{N+1}(x^*) = S(S^N x^*) = Sx^*,$$

which shows that Sx^* is another fixed point of S^N. But as S^N has a unique fixed point, which is x^*, then $S(x^*) = x^*$. The remaining part of the proof follows from Theorem 2.5. □

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