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A GENERAL COMMON FIXED POINT THEOREM FOR MULTI-MAPS SATISFYING AN IMPLICIT RELATION ON FUZZY METRIC SPACES

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Abstract

In this paper, we give a common fixed point theorem for multi-valued mappings satisfying an implicit relation on fuzzy metric spaces.

1 Introduction and Preliminaries

The theory of fuzzy sets was introduced by L.Zadeh [21] in 1965.George and Veeramani [5] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [10].Grabiec[6] proved the contraction principle in the setting of fuzzy metric spaces introduced in [10].For fixed point
theorems in fuzzy metric spaces some of the interesting references are [2-4,6,7,11,14, 15,17-20].Mishra et.al[11] and Cho et.al[3] proved some common fixed point theorems for four single valued self maps on fuzzy metric spaces using a special type of contractive condition. In this paper we prove a common fixed point theorem for four maps of which two are multi valued satisfying the same type of contraction condition under implicit relation without using the following condition

$$\lim_{t\to\infty} M(x,y,t) = 1$$

for all x, y in X.

Definition 1.1. *A binary operation* $* : [0,1]\times[0,1] \longrightarrow [0,1]$ *is a continuous t-norm if it satisfies the following conditions*

1. $*$ *is associative and commutative,*

2. $*$ *is continuous,*

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3. $a * 1 = a$ *for all* $a \in [0, 1]$,

4. $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$, *for each* $a, b, c, d \in [0, 1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = min\{a, b\}$.

Definition 1.2 ([5]). A 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each $x, y, z \in X$ and each t and $s > 0$,

1. $M(x, y, t) > 0$,
2. $M(x, y, t) = 1$ if and only if $x = y$,
3. $M(x, y, t) = M(y, x, t)$,
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
5. $M(x, y, .) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Lemma 1.3 ([6]). Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t, for all x, y in X.

Definition 1.4. Let $(X, M, *)$ be a fuzzy metric space. M is said to be continuous on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$, i.e., whenever

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \text{ and } \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 1.5 (Proposition 1 of [13]). Let $(X, M, *)$ be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.

Lemma 1.6 . Let $(X, M, *)$ be a fuzzy metric space. If we define $E_{\lambda,M} : X^2 \to \mathbb{R}^+ \cup \{0\}$ by

$$E_{\lambda,M}(x, y) = \inf\{t \ : \ M(x, y, t) > 1 - \lambda\}$$

for each $\lambda \in (0, 1)$ and $x, y \in X$, then we have

(i) For any $\mu \in (0, 1)$ there exists $\lambda \in (0, 1)$ such that

$$E_{\mu,M}(x_1, x_n) \leq E_{\lambda,M}(x_1, x_2) + E_{\lambda,M}(x_2, x_3) + \cdots + E_{\lambda,M}(x_{n-1}, x_n)$$

$for \ \ any \ \ x_1, x_2, ..., x_n \in X.$

(ii) The sequence $\{x_n\}$ is convergent in fuzzy metric space $(X, M, *)$ if and only if $E_{\lambda,M}(x_n, x) \to 0$. Also the sequence $\{x_n\}$ is Cauchy sequence if and only if it is Cauchy with $E_{\lambda,M}$.

Proof. *(i)* For every $\mu \in (0,1)$, we can find a $\lambda \in (0,1)$ such that

$$\overbrace{(1-\lambda) * (1-\lambda) * \cdots * (1-\lambda)}^{n} \geq 1-\mu$$

by triangular inequality we have

$$\begin{aligned}
&M(x_1, x_n, E_{\lambda,M}(x_1,x_2) + E_{\lambda,M}(x_2,x_3) + \cdots + E_{\lambda,M}(x_{n-1},x_n) + n\delta) \\
&\geq M(x_1, x_2, E_{\lambda,M}(x_1,x_2) + \delta) * \cdots * M(x_{n-1}, x_n, E_{\lambda,M}(x_{n-1},x_n) + \delta) \\
&\geq \overbrace{(1-\lambda) * (1-\lambda) * \cdots * (1-\lambda)}^{n} \geq 1-\mu
\end{aligned}$$

for every $\delta > 0$, which implies that

$$E_{\mu,M}(x_1,x_n) \leq E_{\lambda,M}(x_1,x_2) + E_{\lambda,M}(x_2,x_3) + \cdots + E_{\lambda,M}(x_{n-1},x_n) + n\delta.$$

Since $\delta > 0$ is arbitrary, we have

$$E_{\mu,M}(x_1,x_n) \leq E_{\lambda,M}(x_1,x_2) + E_{\lambda,M}(x_2,x_3) + \cdots + E_{\lambda,M}(x_{n-1},x_n).$$

(ii). Note that since M is continuous in its third place and

$$E_{\lambda,M}(x,y) = \inf\{t \ : \ M(x,y,t) > 1-\lambda\},$$

we have

$$M(x_n, x, \eta) > 1-\lambda \Longleftrightarrow E_{\lambda,M}(x_n,x) < \eta$$

for every $\eta > 0$. □

Lemma 1.7 . Let (X,M,*) be a fuzzy metric space. If sequence $\{x_n\}$ in X exists such that for every $n \in \mathbb{N}$,

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, k^n t)$$

for every $k > 1$, then the sequence $\{x_n\}$ is a Cauchy sequence.

Proof. For every $\lambda \in (0,1)$ and $x_n, , x_{n+1} \in X$, we have

$$\begin{aligned}
E_{\lambda,M}(x_{n+1},x_n) &= \inf\{t \ : \ M(x_{n+1},x_n,t) > 1-\lambda\} \\
&\leq \inf\{t \ : \ M(x_0,x_1,k^n t) > 1-\lambda\} \\
&= \inf\{\frac{t}{k^n} \ : \ M(x_0,x_1,t) > 1-\lambda\} \\
&= \frac{1}{k^n}\inf\{t \ : \ M(x_0,x_1,t) > 1-\lambda\} \\
&= \frac{1}{k^n}E_{\lambda,M}(x_0,x_1).
\end{aligned}$$

By Lemma 1.6, for every $\mu \in (0,1)$ there exists $\lambda \in (0,1)$ such that

$$E_{\mu,M}(x_n, x_m) \leq E_{\lambda,M}(x_n, x_{n+1}) + E_{\lambda,M}(x_{n+1}, x_{n+2}) + \cdots + E_{\lambda,M}(x_{m-1}, x_m)$$

$$\leq \frac{1}{k^n} E_{\lambda,M}(x_0, x_1) + \frac{1}{k^{n+1}} E_{\lambda,M}(x_0, x_1) + \cdots + \frac{1}{k^{m-1}} E_{\lambda,M}(x_0, x_1)$$

$$= E_{\lambda,M}(x_0, x_1) \sum_{j=n}^{m-1} \frac{1}{k^j} \longrightarrow 0.$$

Hence sequence $\{x_n\}$ is Cauchy . □

Throughout this paper, $CB(X)$ is the set of all non-empty closed and bounded subsets of X. For $A, B \in CB(X)$ and for every $t > 0$,denote

$$\mathcal{M}(A, B, t) = sup\{M(a, b, t); a \in A, b \in B\}$$

and

$$\delta_M(A, B, t) = inf\{M(a, b, t); a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta_M(A, B, t) = \delta_M(a, B, t)$. If B also consists of a single point b, we write $\delta_M(A, B, t) = M(a, b, t)$.

It follows immediately from the definition that

$$\begin{aligned} \delta_M(A, B, t) &= \delta_M(B, A, t) \geq 0, \\ \delta_M(A, B, t) &= 1 \Longleftrightarrow A = B = \{a\}, \\ for \ \ all \ \ A, B \ \ in \ \ CB(X). \end{aligned}$$

The following definition was given by Jungck and Rhoades [9].

Definition 1.8. The mappings $I : X \longrightarrow X$ and $F : X \longrightarrow CB(X)$ are weakly compatible if they commute at coincidence points, i.e., for each point u in X such that $Fu = \{Iu\}$, we have $FIu = IFu$.

Implicit relations on metric spaces have been used in many articles ([1, 8, 12, 16, 18]).

Let $\mathcal{T}$ be the set of all continuous functions $T : [0,1]^5 \longrightarrow [-1,1]$ satisfying the following conditions:

(T_1): $T(t_1, \cdots, t_5)$ is increasing in t_1 and decreasing in $t_2, \cdots, t_5$.

(T_2):$T(u, v, v, v, v) \geq 0$ implies that $u > v$, $\forall v \in [0,1)$ *and* $\forall u \in [0,1]$.

Remark 1.9. It easy to see that $T(v, v, v, v, v) \geq 0$ implies that $v = 1$. If $v \neq 1$, by (T_2) , $T(v, v, v, v, v) \geq 0$ implies that $v > v$, is a contradiction. Thus $v = 1$.

Example 1.10. Let $T : [0,1]^5 \longrightarrow [-1,1]$, be defined by $T(t_1, t_2, t_3, t_4, t_5) = t_1 - (min\{t_2, t_3, t_4, t_5\})^h$ for some $0 < h < 1$.

2 THE MAIN RESULT

Now we give our main theorem.

Theorem 2.1. *Let F, G be mappings of a complete fuzzy metric space $(X, M, *)$ with $t * t = t$ for all $t \in [0,1]$ into $CB(X)$.Also f, g be mappings of X into itself satisfying:*

(i) $Fx \subseteq g(X),\ Gx \subseteq f(X)$ *for every* $x \in X$,

(ii) *The pairs (F, f) and (G, g) are weakly compatible,*

(iii)*there exists a constant $k \in (0,1)$ such that*

$$T\left(\begin{array}{l} \delta_M(Fx, Gy, kt), M(fx, gy, t), \mathcal{M}(fx, Fx, t), \mathcal{M}(gy, Gy, t), \\ \mathcal{M}(fx, Gy, \alpha t) * \mathcal{M}(gy, Fx, (2-\alpha)t) \end{array}\right) \geq 0.$$

for every x, y in X , for every $t > 0$ and $\alpha \in (0,2)$, where $T \in \mathcal{T}$. Suppose that one of $g(X)$ and $f(X)$ is a closed subset of X, then there exists a unique $p \in X$ such that $\{p\} = \{fp\} = \{gp\} = Fp = Gp$.

Proof. Let x_0 be an arbitrary point in X. By (i), we choose a point x_1 in X such that $y_0 = gx_1 \in Fx_0$. For this point x_1 there exists a point x_2 in X such that $y_1 = fx_2 \in Gx_1$, and so on. Continuing in this manner we can define sequences $\{x_n\}$ and $\{y_n\}$ as follows

$$y_{2n} = gx_{2n+1} \in Fx_{2n}, \quad y_{2n+1} = fx_{2n+2} \in Gx_{2n+1},$$

for $n = 0, 1, 2, \cdots$.
Let $d_m(t) = M(y_m, y_{m+1}, t), t > 0$.
Step 1: Putting $x = x_{2n}, y = x_{2n+1}$ in (iii) we have

$$T\left(\begin{array}{c} \delta_M(Fx_{2n}, Gx_{2n+1}, kt), M(fx_{2n}, gx_{2n+1}, t), \\ \mathcal{M}(fx_{2n}, Fx_{2n}, t), \mathcal{M}(gx_{2n+1}, Gx_{2n+1}, t), \\ \mathcal{M}(fx_{2n}, Gx_{2n+1}, \alpha t) * \mathcal{M}(gx_{2n+1}, Fx_{2n}, (2-\alpha)t) \end{array}\right) \geq 0$$

From (T_1),

$$T\left(\begin{array}{l} M(y_{2n}, y_{2n+1}, kt), M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t), \\ M(y_{2n}, y_{2n+1}, t), M(y_{2n-1}, y_{2n+1}, \alpha t) * M(y_{2n}, y_{2n}, (2-\alpha)t) \end{array}\right) \geq 0.$$

Put $\alpha = 1 + q_1$, where $q_1 \in (k, 1)$. Since

$$M(y_{2n-1}, y_{2n+1}, (1+q_1)t) \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n+1}, q_1 t),$$

and T is decreasing in t_5, we get

$$T(d_{2n}(kt), d_{2n-1}(t), d_{2n-1}(t), d_{2n}(t), d_{2n-1}(t) * d_{2n}(q_1 t)) \geq 0. \tag{1}$$

If $d_{2n}(t) < d_{2n-1}(t)$, then since $d_{2n}(q_1 t) * d_{2n-1}(t) \geq d_{2n}(q_1 t) * d_{2n}(q_1 t) = d_{2n}(q_1 t)$ and from (T_1)in inequality (1), we have

$$T(d_{2n}(kt), d_{2n}(q_1 t), d_{2n}(q_1 t), d_{2n}(q_1 t), d_{2n}(q_1 t)) \geq 0.$$

From (T_2) we have $d_{2n}(kt) > d_{2n}(q_1 t)$. It is a contradiction.
Hence $d_{2n}(t) \geq d_{2n-1}(t)$ for every $n \in \mathbb{N}$ and $\forall t > 0$. Now from (1) and (T_1)we have

$$T(d_{2n}(kt), d_{2n-1}(q_1 t), d_{2n-1}(q_1 t), d_{2n-1}(q_1 t), d_{2n-1}(q_1 t)) \geq 0. \tag{2}$$

Step 2: Putting $x = x_{2n}, y = x_{2n-1}$ and $\alpha = 1 - q_2$ where $q_2 \in (k, 1)$ in (iii) we can show that

$$T(d_{2n-1}(kt), d_{2n-2}(q_2 t), d_{2n-2}(q_2 t), d_{2n-2}(q_2 t), d_{2n-2}(q_2 t)) \geq 0. \tag{3}$$

Let $q = min\{q_1, q_2\}$.Then $q \in (k, 1)$ and from (2), (3), (T_1) we have

$$T(d_n, (kt), d_{n-1}(qt), d_{n-1}(qt), d_{n-1}(qt), d_{n-1}(qt)) \geq 0.$$

From (T_2),we have $d_n(kt) \geq d_{n-1}(qt)$, for every $n \in \mathbb{N}$. That is,

$$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, \frac{q}{k}t) \geq \cdots \geq M(y_0, y_1, (\frac{q}{k})^n t).$$

Hence by Lemma 1.7 $\{y_n\}$ is Cauchy and the completeness of X, $\{y_n\}$ converges to p in X. Thus

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} gx_{2n+1} = p \in \lim_{n\to\infty} Fx_{2n},$$

and

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} fx_{2n+2} = p \in \lim_{n\to\infty} Gx_{2n+1}.$$

Suppose that $g(X)$ is closed . Then for some $v \in X$ we have $p = gv \in g(X)$.
Step 3: Putting $x = x_{2n}, y = v$ and $\alpha = 1$ in (iii) we get

$$T\left(\begin{array}{l} \delta_M(Fx_{2n}, Gv, kt), M(fx_{2n}, gv, t), \mathcal{M}(fx_{2n}, Fx_{2n}, t), \\ \mathcal{M}(gv, Gv, t), \mathcal{M}(fx_{2n}, Gv, t) * \mathcal{M}(gv, Fx_{2n}, t) \end{array}\right) \geq 0.$$

By (T_1), we have

$$T\left(\begin{array}{l} \delta_M(y_{2n}, Gv, kt), M(y_{2n-1}, gv, t), M(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(gv, Gv, t), \mathcal{M}(y_{2n-1}, Gv, t) * M(gv, y_{2n}, t) \end{array}\right) \geq 0.$$

On making $n \longrightarrow \infty$ we have

$$T(\delta_M(p, Gv, kt), M(p, gv, t), M(p, p, t), \mathcal{M}(p, Gv, t), \mathcal{M}(p, Gv, t)$$
$$*M(p, p, t)) \geq 0.$$

Thus by (T_1) we get,

$$T(\delta_M(p, Gv, kt), 1, 1, \delta_M(p, Gv, t), \delta_M(p, Gv, t) * 1) \geq 0.$$

Since T is increasing in t_1 and decreasing in $t_2, ..., t_5$,we get

$$T(\delta_M(p, Gv, t), \delta_M(p, Gv, t), \delta_M(p, Gv, t), \delta_M(p, Gv, t), \delta_M(p, Gv, t)) \geq 0.$$

Thus by Remark1.9, we have $\delta_M(p, Gv, t) = 1$. Hence $Gv = \{p\} = \{gv\}$. Since (G, g) is weakly compatible pair we have $Ggv = gGv$, hence $Gp = \{gp\}$.
Step 4: Putting $x = x_{2n}, y = p$ and $\alpha = 1$ in (iii) we get

$$T\left(\begin{array}{l} \delta_M(Fx_{2n}, Gp, kt), M(fx_{2n}, gp, t), \mathcal{M}(fx_{2n}, Fx_{2n}, t), \\ \mathcal{M}(gp, Gp, t), \mathcal{M}(fx_{2n}, Gp, t) * \mathcal{M}(gp, Fx_{2n}, t) \end{array} \right) \geq 0.$$

By (T_1), we have

$$T\left(\begin{array}{l} M(y_{2n}, gp, kt), M(y_{2n-1}, gp, t), M(y_{2n-1}, y_{2n}, t), \\ M(gp, gp, t), M(y_{2n-1}, gp, t) * M(gp, y_{2n}, t) \end{array} \right) \geq 0.$$

On making $n \longrightarrow \infty$, we get

$$T(M(p, gp, kt), M(p, gp, t), M(p, p, t), M(gp, gp, t), M(p, gp, t) * M(gp, p, t)) \geq 0.$$

Thus,

$$T(M(p, gp, t), M(p, gp, t), M(p, gp, t), M(p, gp, t), M(p, gp, t)) \geq 0,$$

by Remark 1.9, we have $M(p, gp, t) = 1$, hence $gp = p$. Therefore, $Gp = \{p\}$.
Step 5: Since $Gp \subseteq f(X)$,there exists $w \in X$ such that $\{fw\} = Gp = \{gp\} = \{p\}$.
Putting $x = w, y = p$ and $\alpha = 1$ in (iii) we get

$$T\left(\begin{array}{l} \delta_M(Fw, Gp, kt), M(fw, gp, t), \mathcal{M}(fw, Fw, t), \\ \mathcal{M}(gp, Gp, t), \mathcal{M}(fw, Gp, t) * \mathcal{M}(gp, Fw, t) \end{array} \right) \geq 0.$$

Thus we have
$$T(\delta_M(Fw, p, kt), M(p, p, t), \mathcal{M}(p, Fw, t), M(p, p, t), M(p, p, t) * \mathcal{M}(p, Fw, t)) \geq 0.$$

Hence by (T_1), we get

$$T(\delta_M(Fw, p, t), \delta_M(Fw, p, t), \delta_M(p, Fw, t), \delta_M(Fw, p, t), \delta(p, Fw, t)) \geq 0.$$

So again by Remark 1.9 we have $\delta_M(Fw, p, t) = 1$. Hence $Fw = \{p\}$. Since $Fw = \{fw\}$ and the pair $\{F, f\}$ isweakly compatible, we obtain $Fp = Ffw = fFw = \{fp\}$.
Step 6: Putting $x = p, y = x_{2n+1}$ and $\alpha = 1$ in (iii) we can show as in
Step 4 that $fp = p$ so that $Fp = \{fp\} = \{p\}$.
Thus $Fp = Gp = \{fp\} = \{gp\} = \{p\}$. Uniqueness of common fixed point follows easily from (iii). Similarly the theorem follows when $f(X)$ is closed. □

Corollary 2.2. *Let F, G be mappings of a complete fuzzy metric space $(X, M, *)$ with $t * t = t$ into $CB(X)$ for all $t \in [0, 1]$.Also f, g be mappings of X into itself satisfying:*

(i) $Fx \subseteq g(X),\ Gx \subseteq f(X)$ *for every* $x \in X$,

(ii) *The pairs (F, f) and (G, g) are weakly compatible,*

(iii) *there exists a constant $k \in (0, 1)$ such that*

$$\delta_M(Fx, Gy, kt) \geq \left(\min \left\{ \begin{array}{c} M(fx, gy, t), \mathcal{M}(fx, Fx, t), \mathcal{M}(gy, Gy, t), \\ \mathcal{M}(fx, Gy, \alpha t) * \mathcal{M}(gy, Fx, (2-\alpha)t) \end{array} \right\} \right)^h$$

for every x, y in X ,for every $t > 0$, $\alpha \in (0, 2)$ and $0 < h < 1$. Suppose that one of $g(X)$ and $f(X)$ is a closed subset of X, then there exists a unique $p \in X$ such that $\{p\} = \{fp\} = \{gp\} = Fp = Gp$.

Proof. The Corollary follows easily from Theorem 2.1 , if we define $T(t_1, t_2, t_3, t_4, t_5) = t_1 - (\min\{t_2, t_3, t_4, t_5\})^h$ in Theorem 2.1, where $0 < h < 1$. □

Now we give the following Corollaries when F and G are also single valued mappings.

Corollary 2.3. *Let $(X, M, *)$ be a complete fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$.Also let F, G, f, g be mappings of X into itself satisfying:*

(i) $F(X) \subseteq g(X),\ G(X) \subseteq f(X)$,

(ii) *The pairs (F, f) and (G, g) are weakly compatible,*

(iii) *there exists a constant $k \in (0, 1)$ such that*

$$M(Fx, Gy, kt) \geq \left(\min \left\{ \begin{array}{c} M(fx, gy, t), M(fx, Fx, t), M(gy, Gy, t), \\ M(fx, Gy, \alpha t) * M(gy, Fx, (2-\alpha)t) \end{array} \right\} \right)^h$$

for every x, y in X , for every $t > 0$, $\alpha \in (0, 2)$ and $0 < h < 1$. Suppose that one of $g(X)$ and $f(X)$ is a closed subset of X, then there exists a unique $p \in X$ such that $p = fp = gp = Fp = Gp$.

Corollary 2.4. *Let $(X, M, *)$ be a complete fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$.Also F, G, f, g be mappings of X into itself satisfying:*

(i) $F(X) \subseteq g(X),\ G(X) \subseteq f(X)$,

(ii) *The pairs (F, f) and (G, g) are weakly compatible,*

(iii) *there exists a constant $k \in (0, 1)$ such that*

$$M(Fx, Gy, kt) \geq (M(fx, gy, t)^h$$

for every x, y in X , for every $t > 0$ and $0 < h < 1$. Suppose that one of $g(X)$ and $f(X)$ is a closed subset of X, then there exists a unique $p \in X$ such that $p = fp = gp = Fp = Gp$.

Proof. The Corollary follows easily if we define $T(t_1, t_2, t_3, t_4, t_5) = t_1 - {t_2}^h$ in Theorem 2.1, where $0 < h < 1$. $\square$

Now we give an example to illustrate our main Theorem 2.1.

Example 2.5. *Let $(X, M, *)$ be a fuzzy metric space, in which $X = [0, 1]$, $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ and*

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $t > 0$.
Define the maps F, G, f, g on X as follows: $Fx = Gx = \{1\}$ and $fx = \frac{x+1}{2}$, $gx = \frac{2x+1}{3}$ for all $x \in X$. Define $T(t_1, t_2, t_3, t_4, t_5) = t_1 - (\min\{t_2, t_3, t_4, t_5\})^h$. Then for any $h, k \in (0, 1)$, the inequality

$$M(Fx, Gy, kt) \geq \left(\min \left\{ \begin{array}{c} M(fx, gy, t), M(fx, Fx, t), M(gy, Gy, t), \\ M(fx, Gy, \alpha t) * M(gy, Fx, (2 - \alpha)t) \end{array} \right\} \right)^h$$

is satisfied for all x, y in X , for every $t > 0$ and for every $\alpha \in (0, 2)$, since the L.H.S. of the inequality is 1. Clearly all conditions in Theorem 2.1 are satisfied. Also 1 is the unique common fixed point of F, G, f and g.

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