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ON *GO*-COMPACT SPACES

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Abstract

The purpose of this paper is to offer some more properties of GO-compact spaces and to introduce and investigate some properties of g-continuous multifunctions. We also investigate GO-compact spaces in the context of multifunctions.

1 Introduction and preliminaries

Levine [11] introduced the concept of generalized closed sets of a topological space. Since the advent of these notions, several research papers with interesting results in different respects came to existence (see, [1], [3], [4], [5], [6], [8], [9], [10], [12]). Recently Caldas and Jafari [4] introduced and investigated the concepts of g-US spaces, g-convergency, sequential GO-compactness, sequential g-continuity and sequential g-sub-continuity.

Throughout the present paper (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by $Int(A)$ and $Cl(A)$, respectively. $A \subset X$ is called a generalized closed set (briefly g-closed set) of X [11] if $Cl(A) \subset G$ holds whenever $A \subset G$ and G is open in X. The union of two g-closed sets is a g-closed set. A subset A of X is called a g-open set of X, if its complement A^c is g-closed in X. The intersection of all g-closed sets containing a set A is called the g-closure of A [10] and is denoted by $gCl(A)$. If $A \subset X$, then $A \subset gCl(A) \subset Cl(A)$. The collection of all g-closed (resp. g-open) subsets of X will be denoted by $GC(X)$ (resp. $GO(X)$). We set $GC(X,x) = \{V \in GC(X)/x \in V\}$ for $x \in X$. We define similarly $GO(X,x)$. Let p be a point of X and N be a subset of X. N is called a g-neighborhood of p in X [3] if there exists a g-open set O of X such that $p \in O \subset N$.

A space X is GO-compact if every g-open cover of X has a finite subcover. Since every open sets is a g-open set, it follows that every GO-compact space is compact.

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But, the converse may be false. Let $X = \{x\} \cup \{x_i : i \in I\}$ where the indexed set I is uncountable. Let $\tau = \{\emptyset, \{x\}, X\}$ be the topology on X. Evidently, X is a compact space. However, it is not a GO-compact space because $\{\{x, x_i\} : i \in I\}$ is a g-open covering of X but it has no finite subcover. A subset A of a space X is said to be GO-compact if A is GO-compact as a subspace of X. If the product space of two no-empty spaces is GO-compact, then each factor space is GO-compact [1]. If A is g-open in X and B is g-open in Y, then $A \times B$ is g-open in $X \times Y$ [11]. A function $f : X \to Y$ is said to be g-continuous [1] if the inverse image of every closed set in Y is g-closed in X.

It is the purpose of this paper to offer some more characterizations of GO-compact spaces. We also introduce the notion of g-complete accumulation points by which we give some characterizations of GO-compact spaces. By introducing the notion of 1-lower (resp. 1-upper) g-continuous functions and considering the known notion of 1-lower (resp. 1-upper) compatible partial orders we investigate some more properties of GO-compactness. We also investigate GO-compact spaces in the context of multifunctions by introducing 1-lower(resp. 1-upper) g-continuous multifunctions. Lastly we also obtain some characterizations of GO-compact spaces by using lower (resp. upper) g-continuous multifunctions. In this paper we are working in ZFC.

Recall that a function $f : X \to Y$ is said to be g-continuous [3] if the inverse image of each open set in Y is g-open in X.

Let Λ be a directed set. Now we introduce the following notions which will be used in this paper. A net $\xi = \{x_\alpha \mid \alpha \in \Lambda\}$ *g-accumulates* at a point $x \in X$ if the net is frequently in every $U \in GO(X, x)$, i.e. for each $U \in GO(X, x)$ and for each $\alpha_0 \in \Lambda$, there is some $\alpha \geq \alpha_0$ such that $x_\alpha \in U$. The net ξ *g-converges* to a point x of X if it is eventually in every $U \in gO(X, x)$. We say that a filterbase $\Theta = \{F_\alpha \mid \alpha \in \Gamma\}$ *g-accumulates* at a point $x \in X$ if $x \in \bigcap_{\alpha\in\Gamma} gCl(F_\alpha)$. Given a set S with $S \subset X$, a *g-cover* of S is a family of g-open subsets U_α of X for each $\alpha \in I$ such that $S \subset \bigcup_{\alpha\in I} U_\alpha$. A filterbase $\Theta = \{F_\alpha \mid \alpha \in \Gamma\}$ *g-converges* to a point x in X if for each $U \in GO(X, x)$, there exists an F_α in Θ such that $F_\alpha \subset U$.

Recall that a multifunction (also called multivalued function [2]) F on a set X into a set Y, denoted by $F : X \to Y$, is a relation on X into Y, i.e. $F \subset X \times Y$. Let $F : X \to Y$ be a multifunction. The upper and lower inverse of a set V of Y are denoted by $F^+(V)$ and $F^-(V)$:

$$F^+(V) = \{x \in X \mid F(x) \subset V\} \text{ and } F^-(V) = \{x \in X \mid F(x) \cap V \neq \emptyset\}.$$

2 *GO*-compact spaces

We begin with the following notions:

Definition 1 *A point x in a space X is said to be a g-complete accumulation point of a subset S of X if $Card(S \cap U) = Card(S)$ for each $U \in GO(X, x)$, where*

$Card(S)$ denotes the cardinality of S.

Example 2.1 *Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{a, b, c\}\}$. Observe that both b and c are g-complete accumulation points of $\{a, b\}$. Notice that a is not a g-complete accumulation point of $\{a, b\}$*

Definition 2 *In a topological space X, a point x is said to be a g-adherent point of a filterbase Θ on X if it lies in the g-closure of all sets of Θ.*

Observe that the Frechet filter does not satisfy Definition 2. But take a topological space X such that $A \subset X$. Then any point of the g-closure of A is a g-adherent point of $\Omega = \{U \subset X \mid A \subset U\}$.

Theorem 2.2 *A space X is GO-compact if and only if each infinite subset of X has a g-complete accumulation point.*

Proof. Let the space X be *GO*-compact and S an infinite subset of X. Let K be the set of points x in X which are not g-complete accumulation points of S. Now it is obvious that for each point x in K, we are able to find $U(x) \in GO(X, x)$ such that $Card(S \cap U(x)) \neq Card(S)$. If K is the whole space X, then $\Theta = \{U(x) \mid x \in X\}$ is a g-cover of X. By the hypothesis X is *GO*-compact, so there exists a finite subcover $\Psi = \{U(x_i)\}$, where $i = 1, 2, ..., n$ such that $S \subset \bigcup\{U(x_i) \cap S \mid i = 1, 2, ..., n\}$. Then $Card(S) = max\{Card(U(x_i) \cap S) \mid i = 1, 2, ..., n\}$ which does not agree with what we assumed. This implies that S has a g-complete accumulation point. Now assume that X is not *GO*-compact and that every infinite subset $S \subset X$ has a g-complete accumulation point in X. It follows that there exists a g-cover Ξ with no finite subcover. Set $\delta = min\{Card(\Phi) \mid \Phi \subset \Xi$, where Φ is a g-cover of $X\}$. Fix $\Psi \subset \Xi$ for which $Card(\Psi) = \delta$ and $\bigcup\{U \mid U \in \Psi\} = X$. Let N denote the set of natural numbers. Then by hypothesis $\delta \geq Card(N)$. By well-ordering of Ψ by some minimal well-ordering " $\sim$ ", suppose that U is any member of Ψ. By minimal well-ordering " $\sim$" we have $Card(\{V \mid V \in \Psi, V \sim U\} < Card(\{V \mid V \in \Psi\})$. Since Ψ can not have any subcover with cardinality less than δ, then for each $U \in \Psi$ we have $X \neq \bigcup\{V \mid V \in \Psi, V \sim U\}$. For each $U \in \Psi$, choose a point $x(U) \in X \setminus \bigcup\{V \cup \{x(V)\} \mid V \in \Psi, V \sim U\}$. We are always able to do this if not one can choose a cover of smaller cardinality from Ψ. If $H = \{x(U) \mid U \in \Psi\}$, then to finish the proof we will show that H has no g-complete accumulation point in X. Suppose that z is a point of the space X. Since Ψ is a g-cover of X then z is a point of some set W in Ψ. By the fact that $U \sim W$, we have $x(U) \in W$. It follows that $T = \{U \mid U \in \Psi$ and $x(U) \in W\} \subset \{V \mid V \in \Psi, V \sim W\}$. But $Card(T) < \delta$. Therefore $Card(H \cap W) < \delta$. But $Card(H) = \delta \geq Card(N)$ since for two distinct points U and W in Ψ, we have $x(U) \neq x(W)$. This means that H has no g-complete accumulation point in X which contradicts our assumptions. Therefore X is *GO*-compact.

Theorem 2.3 *For a space X the following statements are equivalent:*
(1) X is GO-compact;
(2) Every net in X with a well-ordered directed set as its domain g-accumulates to some point of X.

Proof. (1) ⇒ (2): Suppose that X is GO-compact and $\xi = \{x_\alpha \mid \alpha \in \Lambda\}$ a net with a well-ordered directed set Λ as domain. Assume that ξ has no g-adherent point in X. Then for each point x in X, there exist $V(x) \in GO(X, x)$ and an $\alpha(x) \in \Lambda$ such that $V(x) \cap \{x_\alpha \mid \alpha \geq \alpha(x)\} = \emptyset$. This implies that $\{x_\alpha \mid \alpha \geq \alpha(x)\}$ is a subset of $X \setminus V(x)$. Then the collection $C = \{V(x) \mid x \in X\}$ is a g-cover of X. By hypothesis of the theorem, X is GO-compact and so C has a finite subfamily $\{V(x_i)\}$, where $i = 1, 2, ..., n$ such that $X = \bigcup\{V(x_i)\}$. Suppose that the corresponding elements of Λ be $\{\alpha(x_i)\}$, where $i = 1, 2, ..., n$. Since Λ is well-ordered and $\{\alpha(x_i)\}$, where $i = 1, 2, ..., n$ is finite, the largest element of $\{\alpha(x_i)\}$ exists. Suppose it is $\{\alpha(x_l)\}$. Then for $\gamma \geq \{\alpha(x_l)\}$, we have $\{x_\delta \mid \delta \geq \gamma\} \subset \bigcap_{i=1}^{n}(X \setminus V(x_i)) = X \setminus \bigcup_{i=1}^{n} V(x_i) = \emptyset$ which is impossible. This shows that ξ has at least one g-adherent point in X.
(2) ⇒ (1): Now it is enough to prove that each infinite subset has a g-complete accumulation point by utilizing Theorem 3.1. Suppose that $S \subset X$ is an infinite subset of X. According to Zorn's Lemma, the infinite set S can be well-ordered. This means that we can assume S to be a net with a domain which is a well-ordered index set. It follows that S has a g-adherent point z. Therefore z is a g-complete accumulation point of S. This shows that X is GO-compact.

Theorem 2.4 *A space X is GO-compact if and only if each family of g-closed subsets of X with the finite intersection property has a nonempty intersection.*

Proof. Straightforward.

Question 2.5 *Which condition or conditions should be imposed on a topological space X such that the following statements are equivalent:*
(1) X is GO-compact;
(2) Each filterbase on X with at most one g-adherent point is g-convergent.

3 GO-compactness and 1-lower and 1-upper g-continuous functions

In this section we further investigate properties of GO-compactness by 1-lower and 1-upper g-continuous functions. We begin with the following notions and in what follows R denotes the set of real numbers.

Definition 3 *A function $f : X \to R$ is said to be 1-lower (resp. 1-upper) g-continuous at the point y in X if for each $\lambda > 0$, there exists a g-open set $U(y) \in GO(X, y)$ such that $f(x) > f(y) - \lambda$ (resp. $f(x) > f(y) + \lambda$) for every point x in $U(y)$. The function $f$is 1-lower (resp. 1-upper) g-continuous in X if it has these properties for every point x of X.*

Example 3.1 *Take $f : (R, \tau_u) \to (R, \tau)$, where τ_u is the usual topology and τ is the family of sets $\tau = \{(\eta, \infty) \mid \eta \in R\} \cup R$. Such a function is 1-lower g-continuous. Now take $f : (R, \tau_u) \to (R, \sigma)$, where τ_u is the usual topology and σ is the family of sets $\sigma = \{(-\infty, \eta) \mid \eta \in R\} \cup R$. Such a function is 1-upper g-continuous.*

Theorem 3.2 *A function $f : X \to R$ is 1-lower g-continuous if and only if for each $\eta \in R$, the set of all x such that $f(x) \leq \eta$ is g-closed.*

Proof. It is obvious that the family of sets $\tau = \{(\eta, \infty) \mid \eta \in R\} \cup R$ establishes a topology on R. Then the function f is 1-lower g-continuous if and only if $f : X \to (R, \tau)$ is g-continuous. The interval $(-\infty, \eta]$ is closed in (R, τ). It follows that $f^{-1}((-\infty, \eta])$ is g-closed. Therefore the set of all x such that $f(x) \leq \eta$ is equal to $f^{-1}((-\infty, \eta])$ and thus is g-closed.

Corollary 3.3 *A subset S of X is GO-compact if and only if the characteristic function X_S is 1-lower g-continuous.*

Theorem 3.4 *A function $f : X \to R$ is 1-upper g-continuous if and only if for each $\eta \in R$, the set of all x such that $f(x) \geq \eta$ is g-closed.*

Corollary 3.5 *A subset S of X is GO-compact if and only if the characteristic function X_S is 1-upper g-continuous.*

Question 3.6 *Is it true that if the function $G(x) = inf_{i \in I} f_i(x)$ exists, where f_i, are 1-upper g-continuous functions from X into R, then $G(x)$ is 1-upper g-continuous?*

Theorem 3.7 *Let $f : X \to R$ be a 1-lower g-continuous function, where X is GO-compact. Then f assumes the value $m = inf_{x \in X} f(x)$.*

Proof. Suppose $\eta > m$. Since f is 1-lower g-continuous, then the set $K(\eta) = \{x \in X \mid f(x) \leq \eta\}$ is a non-empty g-closed set in X by infimum property. Hence the family $\{K(\eta) \mid \eta > m\}$ is a collection of non-empty g-closed sets with finite intersection property in X. By Theorem 2.4 this family has non-empty intersection. Suppose $z \in \bigcap_{\eta > m} K(\eta)$. Therefore $f(z) = m$ as we wished to prove.

4 GO-compactness and g-continuous multifunctions

In this section, we give some characterizations of GO-compact spaces by using lower (resp. upper) g-continuous multifunctions.

Definition 4 *A multifunction $F : X \to Y$ is said to be lower (resp. upper) g-continuous if $X \setminus F^-(S)$ (resp. $F^-(S)$) is g-closed in X for each open (resp. closed) set S in Y.*

For the following two lemmas we shall assume that if $gCl(A) = A$, then A is g-closed".

Lemma 4.1 *For a multifunction $F : X \to Y$, the following statements are equivalent:*
(1) F is lower g-continuous;

(2) If $x \in F^-(U)$ for a point x in X and an open set $U \subset Y$, then $V \subset F^-(U)$ for some $V \in GO(x)$;
(3) If $x \notin F^+(D)$ for a point x in X and a closed set $D \subset Y$, then $F^+(D) \subset K$ for some g-closed set K with $x \notin K$;
(4) $F^-(U) \in GO(X)$ for each open set $U \subset Y$.

Proof. (1) $\Rightarrow$ (4): Let U be any open set in Y. By (1), $X - F^-(U)$ is g-closed in X and hence $F^-(U) \in GO(X)$
(4) $\Rightarrow$ (2): Let U be any open set of Y and $x \in F^-(U)$. By (4), $F^-(U) \in GO(X)$. Put $V = F^-(U)$. Then $V \in GO(X)$ and $V \subset F^-(U)$.
(2) $\Rightarrow$ (3): Let D be closed in Y and $x \notin F^+(D)$. Then $Y - D$ is open in Y and $x \in X - F^+(D) = F^-(X - D)$. Therefore, There exists $V \in GO(x)$ such that $V \subset F^-(U)$. Now, put $K = X - V$, then $x \notin K$, K is g-closed and $K = X - V \supset X - F^-(Y - D) = F^+(D)$.
(3) $\Rightarrow$ (1): We show that $F^+(H)$ is g-closed for any closed set H of Y. Let H be any closed set and $x \notin F^+(H)$. By (3) there exists a g-closed set K such that $x \notin K$ and $F^+(H) \subset K$; hence $F^+(H) \subset gCl(F^+(H)) \subset K$. Since $x \notin K$, we have $x \notin gCl(F^+(H))$. This implies that $gCl(F^+(H)) \subset F^+(H)$. In general, we have $F^+(H) \subset gCl(F^+(H))$ and hence $F^+(H) = gCl(F^+(H))$. Hence $F^+(H)$ is g-closed for any closed set H of Y.

Lemma 4.2 *For a multifunction $F : X \to Y$, the following statements are equivalent:*
(1) F is upper g-continuous;
(2) If $x \in F^+(V)$ for a point x in X and an open set $V \subset Y$, then $F(U) \subset V$ for some $U \in GO(x)$;
(3) If $x \notin F^-(D)$ for a point x in X and a closed set $D \subset Y$, then $F^-(D) \subset K$ for some g-closed set K with $x \notin K$;
(4) $F^+(U) \in GO(X)$ for each open set $U \subset Y$.

Proof. (1) $\Rightarrow$ (4): Let U be any open set in Y. Then $Y - U$ is closed. By (1), $F^-(Y - U) = X - F^+(U)$ is g-closed in X and hence $F^+(U) \in GO(X)$.
(4) $\Rightarrow$ (2): Let V be any open set of Y and $x \in F^+(V)$. By (4), $F^+(V) \in GO(X)$. Put $U = F^+(V)$. Then $U \in GO(X)$ and $F(U) \subset V$.
(2) $\Rightarrow$ (3): Let D be closed in Y and $x \notin F^-(D)$. Then $Y - D$ is open and $x \in X - F^-(D) = F^+(Y - D)$. By (2), there exists $U \in GO(X)$ such that $F(U) \subset Y - D$. Now, put $K = X - U$, then $x \notin K$, K is g-closed and $K = X - U \supset X - F^+(Y - D) = F^-(D)$.
(3) $\Rightarrow$ (1): We show that $F^-(H)$ is g-closed for any closed set H of Y. Let H be any closed set and $x \notin F^-(H)$. By (3), there exists a g-closed set K such that $x \notin K$ and $F^-(H) \subset K$; hence $F^-(H) \subset gCl(F^-(H)) \subset K$. Since $x \notin K$, we have $x \notin gCl(F^-(H))$. This implies that $gCl(F^-(H)) \subset F^-(H)$. In general, we have $F^-(H) \subset gCl(F^-(H))$ and hence $F^-(H) = gCl(F^-(H))$. Hence $F^-(H)$ is g-closed for any closed set H of Y.

Theorem 4.3 *The following two statements are equivalent for a space X:*
(1) X is GO-compact.

(2) Every lower g-continuous multifunction from X into the closed sets of a space assumes a minimal value with respect to set inclusion relation.

Proof. (1) $\Rightarrow$ (2): Suppose that F is a lower g-continuous multifunction from X into the closed subsets of a space Y. We denote the poset of all closed subsets of Y with the set inclusion relation "$\subseteq$" by Λ. Now we show that $F : X \to \Lambda$ is a lower g-continuous function. We will show that $N = F^-(\{S \subset Y \mid S \in \Lambda \text{ and } S \subseteq C\})$ is g-closed in X for each closed set C of Y. Let $z \notin N$, then $F(z) \neq S$ for every closed set S of Y. It is obvious that $z \in F^-(Y \setminus C)$, where $Y \setminus C$ is open in Y. By Lemma 4.1 (2), we have $W \subset F^-(Y \setminus C)$ for some $W \in GO(z)$. Hence $F(w) \cap (Y \setminus C) \neq \emptyset$ for each w in W. So for each w in W, $F(w) \setminus C \neq \emptyset$. Consequently, $F(w) \setminus S \neq \emptyset$ for every closed subset S of Y for which $S \subseteq C$. We consider that $W \cap N = \emptyset$. This means that N is g-closed. Thus we observe that F assumes a minimal value.

(2) $\Rightarrow$ (1): Suppose that X is not GO-compact. It follows that we have a net $\{x_i \mid i \in \Lambda\}$, where Λ is a well-ordered set with no g-accumulation point by ([8], Theorem 3.2). We give Λ the order topology. Let $M_j = gCl\{x_i \mid i \geq j\}$ for every j in Λ. We establish a multifunction $F : X \to \Lambda$ where $F(x) = \{i \in \Lambda \mid i \geq j_x\}$, j_x is the first element of all those j's for which $x \notin M_j$. Since Λ has the order topology, $F(x)$ is closed. By the fact that $\{j_x \mid x \in X\}$ has no greatest element in Λ, then F does not assume any minimal value with respect to set inclusion. We now show that $F^-(U) \in GO(X)$ for every open set U in Λ. If $U = \Lambda$, then $F^-(U) = X$ which is g-open. Suppose that $U \subset \Lambda$ and $z \in F^-(U)$. It follows that $F(z) \cap U \neq \emptyset$. Suppose $j \in F(z) \cap U$. This means that $j \in U$ and $j \in F(z) = \{i \in \Lambda \mid i \geq j_x\}$. Therefore $M_j \geq M_{j_x}$. Since $z \notin M_{j_x}$, then $z \notin M_j$. There exists $W \in GO(z)$ such that $W \cap \{x_i \mid i \in \Lambda\} = \emptyset$. This means that $W \cap M_j = \emptyset$. Let $w \in W$. Since $W \cap M_j = \emptyset$, it follows that $w \notin M_j$ and since j_w is the first element for which $w \notin M_j$, then $j_w \leq j$. Therefore $j \in \{i \in \Lambda \mid i \geq j_w\} = F(w)$. By the fact that $j \in U$, then $j \in F(w) \cap U$. It follows that $F(w) \cap U \neq \emptyset$ and therefore $w \in F^-(U)$. So we have $W \subset F^-(U)$ and thus $z \in W \subset F^-(U)$. Therefore $F^-(U)$ is g-open. This shows that F is lower g-continuous which contradicts the hypothesis of the theorem. So the space X is GO-compact.

Theorem 4.4 *The following two statements are equivalent for a space X:*
(1) X is GO-compact.
(2) Every upper g-continuous multifunction from X into the subsets of a T_1-space attains a maximal value with respect to set inclusion relation.

Proof. Its proof is similar to that of Theorem 4.3.

The following result concerns the existence of a fixed point for multifunctions on GO-compact spaces.

Theorem 4.5 *Suppose that $F : X \to Y$ is a multifunction from a GO-compact domain X into itself. Let $F(S)$ be g-closed for S being a g-closed set in X. If $F(x) \neq \emptyset$ for every point $x \in X$, then there exists a nonempty, g-closed set C of X such that $F(C) = C$.*

Proof. Let $\Lambda = \{S \subset X \mid S \neq \emptyset, S \in GC(X)$ and $F(S) \subset S\}$. It is evident that x belongs to Λ. Therefore $\Lambda \neq \emptyset$ and also it is partially ordered by set inclusion. Suppose that $\{S_\gamma\}$ is a chain in Λ. Then $F(S_\gamma) \subset S_\gamma$ for each γ. By the fact that the domain is GO-compact and by ([8], Theorem 3.3), $S = \bigcap_\gamma S_\gamma \neq \emptyset$ and also $S \in GC(X)$. Moreover, $F(S) \subset F(S_\gamma) \subset S_\gamma$ for each γ. It follows that $F(S) \subset S_\gamma$. Hence $S \in \Lambda$ and $S = inf\{S_\gamma\}$. It follows from Zorn's lemma that Λ has a minimal element C. Therefore $C \in GC(X)$ and $F(C) \subset C$. Since C is the minimal element of Λ, we have $F(C) = C$.

We close with the following open question:

Question 4.6 *Give a nontrivial example of a GO-compact space?*

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