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A NOTE ON a-OPEN SETS AND e^*-OPEN SETS

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Abstract

The aim of this paper is to investigate some properties of a-open sets and e^*-open sets in topological spaces.

1 Introduction

Some types of sets play an important role in the study of various properties in topological spaces. Many authors introduced and studied various generalized properties and conditions containing some forms of sets in topological spaces. In this paper, we investigate some properties of a-open sets and e^*-open sets. Moreover, the relationships among a-open sets, e^*-open sets and the related classes of sets are investigated.

In this paper, spaces X and Y mean topological spaces. For a subset A of a space X, $Cl(A)$ and $Int(A)$ represent the closure of A and the interior of A, respectively. A subset A of a space X is said to be regular open (resp. regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$) [7]. The δ-interior of a subset A of X is the union of all regular open sets of X contained in A and it is denoted by δ-$Int(A)$ [8]. A subset A is called δ-open if $A = \delta$-$Int(A)$. The complement of a δ-open set is called δ-closed. The δ-closure of a set A in a space (X, τ) is defined by $\{x \in X : A \cap Int(Cl(B)) \neq \emptyset,\ B \in \tau \text{ and } x \in B\}$ and it is denoted by δ-$Cl(A)$.

Definition 1 *A subset A of a space (X, τ) is called*

(1) δ-preopen [6] if $A \subset Int(\delta\text{-}Cl(A))$ and δ-preclosed [6] if $Cl(\delta\text{-}Int(A)) \subset A$,

(2) δ-semiopen [5] if $A \subset Cl(\delta\text{-}Int(A))$ and δ-semiclosed [5] if $Int(\delta\text{-}Cl(A)) \subset A$,

(3) e-open [2] if $A \subset Cl(\delta\text{-}Int(A)) \cup Int(\delta\text{-}Cl(A))$ and e-closed [2] if $Cl(\delta\text{-}Int(A)) \cap Int(\delta\text{-}Cl(A)) \subset A$,

(4) e^*-open [3] if $A \subset Cl(Int(\delta\text{-}Cl(A)))$ and e^*-closed [3] if $Int(Cl(\delta\text{-}Int(A))) \subset A$,

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(5) a-open [4] if $A \subset Int(Cl(\delta\text{-}Int(A)))$ and a-closed [4] if $Cl(Int(\delta\text{-}Cl(A))) \subset A$.

We denote the δ-boundary $\delta\text{-}Cl(A) \backslash \delta\text{-}Int(A)$ of A by $\delta\text{-}Fr(A)$. A subset A of a space X is said to be a δ-dense set if $\delta\text{-}Cl(A) = X$. The family of all δ-semiopen (resp. δ-preopen, e^*-open, a-open) sets of X is denoted by $\delta SO(X)$ (resp. $\delta PO(X)$, $e^*O(X)$, $aO(X)$).

Remark 2 *([4]) The following diagram holds for a subset A of a space X:*

$$\begin{array}{ccccc}
 & & e^*\text{-open} & & \\
 & & \uparrow & & \\
 & & e\text{-open} & & \\
 & \nearrow & & \nwarrow & \\
\delta\text{-semiopen} & \leftarrow & a\text{-open} & \rightarrow & \delta\text{-preopen} \\
 & & \uparrow & & \\
 & & \delta\text{-open} & & \\
 & & \uparrow & & \\
 & & \text{regular open} & &
\end{array}$$

2 a-open sets and e^*-open sets

Theorem 3 *Let N be a subset of a topological space X. The following are equivalent:*
(1) N is regular open,
(2) N is a-open and e^*-closed,
(3) N is δ-preopen and δ-semiclosed.

Proof. (1) $\Rightarrow$ (2) : Obvious.
(2) $\Rightarrow$ (1) : Let N be a-open and e^*-closed. We have $N \subset Int(Cl(\delta\text{-}Int(N)))$ and $Int(Cl(\delta\text{-}Int(N))) \subset N$ and hence $N = Int(Cl(\delta\text{-}Int(N)))$. Thus, N is regular open.
(1) $\Leftrightarrow$ (3) : Let N be δ-preopen and δ-semiclosed. Then $N \subset Int(\delta\text{-}Cl(N))$ and $Int(\delta\text{-}Cl(N)) \subset N$. Thus, $N = Int(\delta\text{-}Cl(N)) = Int(Cl(N))$ and hence N is regular open. The converse is similar. ■

Theorem 4 *Let N be a subset of a topological space X. The following are equivalent:*
(1) N is δ-semiopen,
(2) N is e^*-open and $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$.

Proof. (1) $\Rightarrow$ (2) : Let N be δ-semiopen. We have $Int(\delta\text{-}Cl(N)) \subset \delta\text{-}Cl(N) \subset Cl(\delta\text{-}Int(N))$. Since

$$\delta\text{-}Int(\delta\text{-}Fr(N)) = \delta\text{-}Int(\delta\text{-}Cl(N) \cap (X \backslash \delta\text{-}Int(N))) = \delta\text{-}Int(\delta\text{-}Cl(N)) \backslash Cl(\delta\text{-}Int(N)),$$

then $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$.

(2) $\Rightarrow$ (1) : Let N be e^*-open and $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$. Then $N \subset Cl(Int(\delta\text{-}Cl(N))) \subset Cl(\delta\text{-}Int(N))$. Thus, N is δ-semiopen. ■

Theorem 5 *For a topological space (X, τ), the family of all a-open sets of X forms a topology, denoted by τ_a, for X.*

Proof. It is obvious that $\varnothing$, $X \in aO(X)$ and any union of a-open sets is a-open.

Let A, $B \in aO(X)$. This implies that $A \subset Int(Cl(\delta\text{-}Int(A)))$ and $B \subset Int(Cl(\delta\text{-}Int(B)))$ and hence

$$\begin{aligned} A \cap B &\subset Int(Cl(\delta\text{-}Int(A))) \cap Int(Cl(\delta\text{-}Int(B))) \\ &\subset Int(Int(Cl(\delta\text{-}Int(A))) \cap Cl(\delta\text{-}Int(B))) \\ &\subset Int(Cl(Int(Cl(\delta\text{-}Int(A))) \cap \delta\text{-}Int(B))) \\ &\subset Int(Cl(\delta\text{-}Int(\delta\text{-}Cl(\delta\text{-}Int(A)) \cap \delta\text{-}Int(B)))) \\ &\subset Int(Cl(\delta\text{-}Int(\delta\text{-}Cl(\delta\text{-}Int(A) \cap \delta\text{-}Int(B))))) \\ &= Int(Cl(\delta\text{-}Int(\delta\text{-}Cl(\delta\text{-}Int(A \cap B))))) \\ &= Int(Cl(\delta\text{-}Int(A \cap B))). \end{aligned}$$

Thus, $A \cap B \in aO(X)$. ■

Theorem 6 *Let X be a topological space. Then $aO(X) = \delta SO(X) \cap \delta PO(X)$.*

Proof. Let $N \in aO(X)$. Then $N \in \delta SO(X)$ and $N \in \delta PO(X)$. Thus, $aO(X) \subset \delta SO(X) \cap \delta PO(X)$.

Conversely, let $N \in \delta SO(X) \cap \delta PO(X)$. Then $N \in \delta SO(X)$ and $N \in \delta PO(X)$. Since $N \in \delta SO(X)$, then by Theorem 4, $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$. Since

$$\delta\text{-}Int(\delta\text{-}Fr(N)) = \delta\text{-}Int(\delta\text{-}Cl(N) \cap (X \backslash \delta\text{-}Int(N))) = \delta\text{-}Int(\delta\text{-}Cl(N)) \backslash \delta\text{-}Cl(\delta\text{-}Int(N)),$$

then $Int(\delta\text{-}Cl(N)) \subset Cl(\delta\text{-}Int(N))$. Since $N \in \delta PO(X)$, we have

$$N \subset Int(\delta\text{-}Cl(N)) \subset Int(Cl(\delta\text{-}Int(N))).$$

Thus, $N \in aO(X)$. ■

Theorem 7 *Let N be a subset of a topological space X. The following are equivalent:*

(1) N is δ-clopen,

(2) N is a-open and δ-preclosed,

(3) N is clopen.

Proof. (1)$\Rightarrow$(2): Obvious.

(2)$\Rightarrow$(1): Let N be a-open and δ-preclosed. We have $N \subset Int(Cl(\delta\text{-}Int(N)))$ and $Cl(\delta\text{-}Int(N)) \subset N$ and hence $N \subset Int(Cl(\delta\text{-}Int(N))) \subset Cl(\delta\text{-}Int(N)) \subset N$. Thus, $N = Int(Cl(\delta\text{-}Int(N))) = Cl(\delta\text{-}Int(N))$ and hence N is δ-open and δ-closed.

(1)$\Longleftrightarrow$(3): Obvious. ■

Theorem 8 *Let N be a subset of a topological space X. The following are equivalent:*

(1) N is δ-preopen,

(2) There exists a regular open set $U \subset X$ such that $N \subset U$ and $\delta\text{-}Cl(N) = \delta\text{-}Cl(U)$,

(3) N is the intersection of a regular open set and a δ-dense set,

(4) N is the intersection of a δ-open set and a δ-dense set.

Proof. (1) $\Rightarrow$ (2) : Let N be δ-preopen. We have $N \subset Int(\delta\text{-}Cl(N))$. Take $U = Int(\delta\text{-}Cl(N))$. Then U is regular open such that $N \subset U$ and $\delta\text{-}Cl(N) = \delta\text{-}Cl(U)$.

(2) $\Rightarrow$ (3) : Suppose that there exists a regular open set $U \subset X$ such that $N \subset U$ and $\delta\text{-}Cl(N) = \delta\text{-}Cl(U)$. Put $M = N \cup (X \backslash U)$. Thus, M is δ-dense and $N = U \cap M$.

(3) $\Rightarrow$ (4) : Obvious.

(4) $\Rightarrow$ (1) : Let $N = U \cap M$ such that U is δ-open and M is δ-dense. We have $\delta\text{-}Cl(N) = \delta\text{-}Cl(U)$. Thus,

$$N \subset U \subset \delta\text{-}Cl(U) = \delta\text{-}Cl(N).$$

Thus, $N \subset Int(\delta\text{-}Cl(N))$ and hence N is δ-preopen. ■

Theorem 9 *Let X be a topological space. If $N \in \delta SO(X)$ and $M \in \delta PO(X)$, then $N \cap M \in e^*O(X)$.*

Proof. Let $N \in \delta SO(X)$ and $M \in \delta PO(X)$. Then

$$\begin{aligned} N \cap M \quad & \subset Cl(\delta\text{-}Int(N)) \cap Int(\delta\text{-}Cl(M)) \\ & \subset Cl(\delta\text{-}Int(N) \cap Int(\delta\text{-}Cl(M))) \\ & = Cl(\delta\text{-}Int(\delta\text{-}Int(N) \cap \delta\text{-}Cl(M))) \\ & \subset Cl(\delta\text{-}Int(\delta\text{-}Cl(\delta\text{-}Int(N) \cap M))) \\ & \subset Cl(Int(\delta\text{-}Cl(N \cap M))). \end{aligned}$$

Thus, $N \cap M \in e^*O(X)$. ■

Theorem 10 *Let N be a subset of a topological space X. The following are equivalent:*

(1) $N \in e^*O(X)$,

(2) N is the intersection of a δ-semiopen and a δ-dense set,

(3) N is the intersection of a δ-semiopen and a δ-preopen set.

Proof. (1) $\Rightarrow$ (2) : Let $N \in e^*O(X)$. Then $\delta\text{-}Cl(N)$ is regular closed and so δ-semiopen. Take $M = N \cup (X \backslash \delta\text{-}Cl(N))$. This implies that M is δ-dense and $N = \delta\text{-}Cl(N) \cap M$.

(2) $\Rightarrow$ (3) : Obvious.

(3) $\Rightarrow$ (1) : It follows from Theorem 9. ■

Theorem 11 *([3]) Let N be a subset of a topological space X. The following are equivalent:*
(1) $N \in e^*O(X)$,
(2) there exists $U \in \delta PO(X)$ such that $U \subset N \subset \delta\text{-}Cl(U)$,
(3) $\delta\text{-}Cl(N)$ is regular closed.

Theorem 12 *Let X be a topological space. The following are equivalent:*
(1) $\delta PO(X) \subset \delta SO(X)$,
(2) $aO(X) = \delta PO(X)$,
(3) $e^*O(X) = \delta SO(X)$.

Proof. (1) $\Leftrightarrow$ (2) : It follows from Theorem 6.
(1) $\Rightarrow$ (3) : We have $e^*O(X) \supset \delta SO(X)$.
Let $N \in e^*O(X)$. By Theorem 11, we have $U \subset N \subset \delta\text{-}Cl(U)$ for a $U \in \delta PO(X)$. Since $U \in \delta SO(X)$, then $Cl(\delta\text{-}Int(U)) = \delta\text{-}Cl(U)$. Hence

$$N \subset \delta\text{-}Cl(U) = Cl(\delta\text{-}Int(U)) \subset Cl(\delta\text{-}Int(N)).$$

Thus, $N \in \delta SO(X)$. This implies that $e^*O(X) = \delta SO(X)$.
(3) $\Rightarrow$ (1) : Obvious. ■

Remark 13 *Let X be a topological space. If X is not a disjoint union of two nonempty δ-preopen subsets, then X is connected. The following example shows that this implication is not reversible.*

Example 14 *Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Then (X, τ) is connected but it is a disjoint union of two nonempty δ-preopen subsets.*

Definition 15 *A topological space X is said to be δp-connected [1] if X cannot be expressed as the union of two nonempty disjoint δ-preopen sets of X.*

Theorem 16 *Let X be a topological space. The following are equivalent:*
(1) X is connected and $\delta PO(X) \subset \delta SO(X)$,
(2) X is not a disjoint union of two nonempty δ-preopen subsets.

Proof. (1) $\Rightarrow$ (2) : Suppose that $X = N \cup M$ such that N and M are disjoint δ-preopen sets. Since $\delta PO(X) \subset \delta SO(X)$, then N and M are a-open. By Theorem 7, M and N are δ-clopen. Since X is connected, then $N = \varnothing$ or $M = \varnothing$.
(2) $\Rightarrow$ (1) : Suppose that X is not a disjoint union of two nonempty δ-preopen subsets. Then X is connected.
Let N be a δ-preopen set and $M = N \backslash Cl(\delta\text{-}Int(N))$. Since an intersection of a δ-preopen set and a δ-open set is δ-preopen, then M is δ-preopen. We have

$$\delta\text{-}Int(M) = \delta\text{-}Int(N) \cap (X \backslash Cl(\delta\text{-}Int(N))) = \varnothing.$$

Also, M is δ-preclosed. By (2), either $M = \varnothing$ or $M = X$ and hence either $N \subset Cl(\delta\text{-}Int(N))$ or $N = X$. Hence, every δ-preopen subset is δ-semiopen. ■

3 Further properties

Theorem 17 *Let X be a topological space. The following are equivalent:*
(1) The δ-closure of every δ-open subset of X is δ-open,
(2) $Cl(\delta\text{-}Int(N)) \subset Int(\delta\text{-}Cl(N))$ for every subset N of X,
(3) $\delta SO(X) \subset \delta PO(X)$,
(4) The δ-closure of every e^*-open subset is δ-open,
(5) $e^*O(X) \subset \delta PO(X)$.

Proof. (1) ⇒ (2) : Suppose that the δ-closure of every δ-open subset of X is δ-open. Then the set $Cl(\delta\text{-}Int(N))$ is δ-open. Hence, $Cl(\delta\text{-}Int(N)) = Int(Cl(\delta\text{-}Int(N))) \subset Int(\delta\text{-}Cl(N))$.

(2) ⇒ (3) : Let N be δ-semiopen. By (2), we have $N \subset Cl(\delta\text{-}Int(N)) \subset Int(\delta\text{-}Cl(N))$. Thus, N is δ-preopen.

(3) ⇒ (4) : Let N be e^*-open. Then $\delta\text{-}Cl(N)$ is δ-semiopen. By (3), $\delta\text{-}Cl(N)$ is δ-preopen. Thus, $\delta\text{-}Cl(N) \subset Int(\delta\text{-}Cl(N))$ and hence $\delta\text{-}Cl(N)$ is δ-open.

(4) ⇒ (5) : Let N be e^*-open. By (4), $\delta\text{-}Cl(N) = Int(\delta\text{-}Cl(N))$. Thus, $N \subset \delta\text{-}Cl(N) = Int(\delta\text{-}Cl(N))$ and hence N is δ-preopen.

(5) ⇒ (1) : Let U be δ-open. Then $\delta\text{-}Cl(U)$ is e^*-open. By (5), $\delta\text{-}Cl(U)$ is δ-preopen. Thus, $\delta\text{-}Cl(U) \subset Int(\delta\text{-}Cl(U))$ and hence $\delta\text{-}Cl(U)$ is δ-open. ■

Theorem 18 *Let X be a topological space. The following are equivalent:*
(1) $\delta PO(X) \subset \delta SO(X)$,
(2) every δ-dense subset is δ-semiopen,
(3) $\delta\text{-}Int(N)$ is δ-dense for every δ-dense subset N,
(4) $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$ for every subset N,
(5) $e^*O(X) \subset \delta SO(X)$,
(6) $\delta\text{-}Int(\delta\text{-}Fr(N)) = \varnothing$ for every δ-dense subset N.

Proof. (1) ⇒ (2) : It follows from the fact that every δ-dense set is δ-preopen.

(2) ⇒ (3) : Let N be δ-dense. Then N is δ-semiopen. Thus, $Cl(\delta\text{-}Int(N)) \supset \delta\text{-}Cl(N) = X$ and hence $\delta\text{-}Int(N)$ is δ-dense.

(3) ⇒ (4) : Let $N \subset X$. We have

$$X = \delta\text{-}Cl(N) \cup (X \backslash \delta\text{-}Cl(N) = \delta\text{-}Cl(N) \cup \delta\text{-}Int(X \backslash N).$$

This implies that $N \cup \delta\text{-}Int(X\backslash N)$ is δ-dense. Hence, $\delta\text{-}Int(N \cup \delta\text{-}Int(X\backslash N))$ is δ-dense. We have

$$\delta\text{-}Int[N \cup \delta\text{-}Int(X\backslash N)] \cap \delta\text{-}Int[(X\backslash N) \cup \delta\text{-}Int(N)] = X \backslash \delta\text{-}Fr(N).$$

Since $X\backslash\delta\text{-}Fr(N)$ is an intersection of two δ-dense δ-open sets, then $X\backslash\delta\text{-}Fr(N)$ is δ-dense.

(4) ⇒ (6) : Obvious.

(6) ⇒ (3) : Let N be δ-dense. By (6), $\delta\text{-}Int(\delta\text{-}Fr(N)) = \delta\text{-}Int(X\backslash\delta\text{-}Int(N)) = X\backslash Cl(\delta\text{-}Int(N)) = \varnothing$. Hence, $\delta\text{-}Int(N)$ is δ-dense.

(4) ⇒ (5) : Let N be e^*-open. By (4) and Theorem 4, N is δ-semiopen.

(5) ⇒ (1) : Obvious. ■

Theorem 19 *Let X be a topological space. The following are equivalent:*
(1) $\delta PO(X) \subset \delta SO(X)$,
(2) $Int(\delta\text{-}Cl(N \cap M)) = Int(\delta\text{-}Cl(N)) \cap Int(\delta\text{-}Cl(M))$ for every $N \subset X$ and $M \subset X$,
(3) $Cl(\delta\text{-}Int(N \cup M)) = Cl(\delta\text{-}Int(N)) \cup Cl(\delta\text{-}Int(M))$ for every $N \subset X$ and $M \subset X$.

Proof. (1) $\Rightarrow$ (2) : Let $\delta PO(X) \subset \delta SO(X)$ and N, $M \subset X$. By Theorem 18, $\delta\text{-}Int(\delta\text{-}Fr(A)) = \varnothing$ for every subset A. Since $\delta\text{-}Int(\delta\text{-}Fr(A)) = \delta\text{-}Int(\delta\text{-}Cl(A) \cap (X \backslash \delta\text{-}Int(A))) = \delta\text{-}Int(\delta\text{-}Cl(A)) \backslash \delta\text{-}Cl(\delta\text{-}Int(A))$, then $\delta\text{-}Int(\delta\text{-}Cl(A)) \subset \delta\text{-}Cl(\delta\text{-}Int(A))$ and hence $\delta\text{-}Int(\delta\text{-}Cl(A)) = \delta\text{-}Int(\delta\text{-}Cl(\delta\text{-}Int(A)))$. This implies that

$$\begin{aligned} & Int(\delta\text{-}Cl(N)) \cap Int(\delta\text{-}Cl(M)) \\ & = Int(Cl(\delta\text{-}Int(N)) \cap Int(\delta\text{-}Cl(M))) \subset Cl(\delta\text{-}Int(N)) \cap Int(\delta\text{-}Cl(M)). \end{aligned}$$

On the other we have

$$\begin{aligned} Cl(\delta\text{-}Int(N)) \cap Int(\delta\text{-}Cl(M)) & \subset Cl(\delta\text{-}Int(N) \cap Int(\delta\text{-}Cl(M))) \\ & \subset Cl(\delta\text{-}Int(N) \cap \delta\text{-}Cl(M)) \\ & \subset Cl(\delta\text{-}Int(N) \cap M) \\ & \subset \delta\text{-}Cl(N \cap M). \end{aligned}$$

Since $Int(\delta\text{-}Cl(N \cap M)) \subset Int(\delta\text{-}Cl(N)) \cap Int(\delta\text{-}Cl(M))$, we have $Int(\delta\text{-}Cl(N \cap M)) = Int(\delta\text{-}Cl(N)) \cap Int(\delta\text{-}Cl(M))$.

(2) $\Rightarrow$ (1) : Suppose that (2) holds. This implies that

$$\begin{aligned} \delta\text{-}Int(\delta\text{-}Fr(N)) & = \delta\text{-}Int(\delta\text{-}Cl(N) \cap \delta\text{-}Cl(X \backslash N)) \\ & = Int(\delta\text{-}Cl(N)) \cap Int(\delta\text{-}Cl(X \backslash N)) \\ & = Int(\delta\text{-}Cl(N \cap X \backslash N)) \\ & = \varnothing. \end{aligned}$$

By Theorem 18, we have $\delta PO(X) \subset \delta SO(X)$.

(2) $\Leftrightarrow$ (3) : Obvious. ■

Theorem 20 *Let X be a topological space. The following are equivalent:*
(1) $\delta PO(X) \subset \delta SO(X)$ and the δ-closure of every δ-open subset of X is δ-open,
(2) $Int(\delta\text{-}Cl(N)) = Cl(\delta\text{-}Int(N))$ for every N in X,
(3) $Cl(Int(\delta\text{-}Cl(N)) = Int(Cl(\delta\text{-}Int(N)))$ for every N in X,
(4) $e^*O(X) \subset aO(X)$,
(5) $\delta SO(X) \subset aO(X)$ and $\delta PO(X) \subset aO(X)$,
(6) $\delta PO(X) = \delta SO(X)$,
(7) N is δ-semiopen if and only if $\delta\text{-}Cl(N)$ is δ-open.

Proof. (1) $\Rightarrow$ (2) : It follows from Theorem 17 and 18.

(2) $\Rightarrow$ (3) : Let $N \subset X$. Since $Int(\delta\text{-}Cl(N)) = Cl(\delta\text{-}Int(N))$ is δ-clopen, then $Cl(Int(\delta\text{-}Cl(N)) = Int(Cl(\delta\text{-}Int(N)))$.

(3) $\Rightarrow$ (4) : Let N be e^*-open. We have $N \subset Cl(Int(\delta\text{-}Cl(N)) = Int(Cl(\delta\text{-}Int(N)))$. Hence, N is a-open.

(4) $\Rightarrow$ (5), (5) $\Rightarrow$ (6) : Obvious.

(6) $\Rightarrow$ (7) : Let N be δ-semiopen. This implies that δ-$Cl(N)$ is δ-semiopen and hence δ-preopen. Thus, $\delta\text{-}Cl(N) \subset Int(\delta\text{-}Cl(N))$ and hence δ-$Cl(N)$ is δ-open.

Conversely, let δ-$Cl(N)$ be δ-open. This implies that $N \subset \delta\text{-}Cl(N) = Int(\delta\text{-}Cl(N))$ and N is δ-preopen and hence δ-semiopen.

(7) $\Rightarrow$ (1) : Let N be δ-open. Then δ-$Cl(N)$ is δ-open.

Let N be a δ-dense set. Then δ-$Cl(N)$ is δ-open. By 7, N is δ-semiopen. Thus, by Theorem 18, $\delta PO(X) \subset \delta SO(X)$. ■

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