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# BEST PROXIMITY POINT THEOREMS FOR WEAK CYCLIC KANNAN CONTRACTIONS

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**Abstract**

In this paper we introduce the notion of weak cyclic Kannan contraction. We give some convergence and existence results for best proximity points for weak cyclic Kannan contractions in the setting of a uniformly convex Banach space.

## 1 Introduction

One of the most important result in fixed point theory is the Banach Contraction Mapping Principle, also known as the theorem of Picard-Banach-Caccioppoli. Several extensions of this result have appeared in the literature devoted to nonlinear contractive operators. One of the most interesting extensions was given by Kirk, Srinivasan and Veeramani in [7], as follows:

**Theorem 1.** *([7]) Let $A$ and $B$ be two nonempty closed subsets of a complete metric space, and suppose $T : A \cup B \to A \cup B$ satisfies the following conditions:*

$$T(A) \subseteq B \text{ and } T(B) \subseteq A; \tag{1}$$

*and*

$$d(Tx, Ty) \leq ad(x, y), \text{ for all } x \in A, y \in B. \tag{2}$$

*where $a \in (0, 1)$. Then $T$ has a unique fixed point in $A \cap B$.*

A mapping satisfying (1) is called *cyclic*. In [11, 15], the contractive condition due to Kannan [4], in the cyclical form, was introduced as a *cyclic Kannan contraction*, for $p \in \mathbb{N}, p > 1$ sets. For two sets, $A$ and $B$, we have the following particular result:

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**Theorem 2.** *Let $A$ and $B$ be nonempty closed subsets of a complete metric space $(X, d)$ and suppose $T : A \cup B \to A \cup B$ is a cyclical operator, such that*

$$d(Tx, Ty) \leq a\big[d(x, Tx) + d(y, Ty)\big], \text{for all } x \in A, y \in B, \tag{3}$$

*where $a \in \big[0, \frac{1}{2}\big)$ is a constant. Then*

*(i) $T$ has a unique fixed point $x^*$ in $A \cap B$.*

*(ii) the Picard iteration $\{T^n x_0\}$ converges to $x^*$ for any starting point $x_0 \in A \cup B$;*

*(iii) the following estimates hold*

$$d(T^n x_0, x^*) \leq \frac{\lambda^n}{1-\lambda} d(x_0, Tx_0), n = 0, 1, 2 \dots$$

$$d(T^n x_0, x^*) \leq \frac{\lambda}{1-\lambda} d(T^{n-1} x_0, T^n x_0), n = 1, 2 \dots$$

*where $\lambda = \dfrac{a}{1-a}$;*

*(iv) the rate of convergence of Picard iteration is given by*

$$d(T^n x_0, x^*) \leq \lambda d(T^{n-1} x_0, x^*), n = 1, 2, \dots$$

*where $\lambda = \dfrac{a}{1-a}$.*

For other results regarding cyclical contractive conditions when the intersection of the sets is nonempty, see [10, 12, 13, 14, 15].

In [3], A. Antony Eldred and P. Veeramani extended Theorem 1 to the case when $A \cap B = \emptyset$, and in this case they didn't seek for the existence of a fixed point of $T$ but for the existence of a *best proximity point*, that is, a point $x$ in $A \cup B$ such that

$$\|x - Tx\| = D(A, B),$$

where

$$D(A, B) = \inf\{\|x - y\| : x \in A, y \in B\}.$$

Their main result is the following:

**Theorem 3.** *([3]) Let $A$ and $B$ be nonempty closed convex subsets of a uniformly convex Banach space. Suppose $T : A \cup B \to A \cup B$ is a cyclic mapping, that satisfies (1) and*

$$\|Tx - Ty\| \leq k\|x - y\| + (1 - k)D(A, B), \text{ for all } x \in A, y \in B, \tag{4}$$

*where $k \in (0, 1)$. Then there exists a unique best proximity point $x$ in $A$. Further, if $x_0 \in A$ and $x_{n+1} = Tx_n$, then $\{x_{2n}\}$ converges to the best proximity point.*

Recently this result has been extended to the case of $p > 1$ sets, see [6]. In [9] a new class of maps was introduced, called cyclic $\varphi$-contractions which contains the cyclic contractions maps as a subclass and for this type of contractive conditions, results of best proximity points were obtained. Another generalization of Theorem 3 was obtained by D. Bari, T. Suzuki and C. Vetro in [2] where it was introduced the notion of cyclic Meir-Keller contractions, in the case of two sets. This result was also generalized for $p$ sets by S. Karpagam and S. Agrawal in [5]. Notice that all the mentioned results regarding best proximity points for cyclic mappings are obtained under some contractive conditions which implies nonexpansivity property.

The main purpose of this paper is to introduce a new class of cyclic contractive operators, called weak cyclic Kannan contractions and to give sufficient conditions for the existence of a unique best proximity point of these operator.

## 2 Preliminaries

Inspired by the results in [2] and [3], we introduce the following class of cyclic contractive conditions which extends (3) for the case $A \cap B = \emptyset$.

**Definition 1.** Let $(X, d)$ be a metric space and let $A$ and $B$ be nonempty subsets of $X$. Then a cyclic operator $T : A \cup B \to A \cup B$ is called *weak cyclic Kannan contraction* if it satisfies the following condition

$$d(Tx, Ty) \leq a[d(x, Tx) + d(y, Ty)] + (1 - 2a)D(A, B), \text{ for all } x \in A, y \in B, \quad (5)$$

where $a \in [0, \frac{1}{2})$.

Note that (5) can be rewritten as

$$d(Tx, Ty) - D(A, B) \leq a[d(x, Tx) + d(y, Ty) - 2D(A, B)],$$

for all $x \in A, y \in B$.

**Example 1.** Let $X = l^p$, $1 \leq p \leq \infty$ and $k \in (0, 1)$. Define two sets by

$$A = \{((1 + k^{2n})e_{2n}) : n \in \mathbb{N}\}, B = \{((1 + k^{2m-1})e_{2m-1}) : m \in \mathbb{N}\}.$$

Define an operator $T : A \cup B \to A \cup B$ by

$$T((1 + k^{2n})e_{2n}) = (1 + k^{2n+1})e_{2n+1},$$

and

$$T((1 + k^{2m-1})e_{2m-1}) = (1 + k^{2m})e_{2m}.$$

Then $D(A, B) = 2^{\frac{1}{p}}$ and $T$ is a cyclic contraction, see [3]. Also, $T$ is a weak cyclic Kannan contraction. Indeed, we have:

$$
\begin{aligned}
\Big[ & (1+k^{2n+1})^p + (1+k^{2m})^p\Big]^{\frac{1}{p}} \\
= & \left[\left(1-k+\frac{k^{2n+1}}{2}+\frac{k^{2n+1}}{2}+k\right)^p + \left(1-k+\frac{k^{2m}}{2}+\frac{k^{2m}}{2}+k\right)^p\right]^{\frac{1}{p}} \\
\leq & \left[\left(1-k+\frac{k^{2n+1}}{2}+\frac{k^{2n+2}}{2}+k\right)^p + \left(1-k+\frac{k^{2m}}{2}+\frac{k^{2m+1}}{2}+k\right)^p\right]^{\frac{1}{p}} \\
\leq & \frac{k}{2}\left\{\left[(1+k^{2n})^p + (1+k^{2n+1})^p\right]^{\frac{1}{p}} + \left[(1+k^{2m-1})^p + (1+k^{2m})^p\right]^{\frac{1}{p}}\right\} \\
+ & (1-k)2^{\frac{1}{p}}.
\end{aligned}
$$

Consequently, $T$ is a weak cyclic Kannan contraction with $a := \dfrac{k}{2} \in \left(0, \dfrac{1}{2}\right)$.

**Example 2.** Let $X = \mathbb{R}$ with the usual norm, and

$$0 \leq b \leq \alpha \leq c \leq e \leq \beta \leq f,$$

be real numbers. Let's denote by $A := [b, c]$ and $B := [e, f]$ and define an operator $T : A \cup B \to A \cup B$ by

$$Tx = \begin{cases} \beta & \text{, if } b \leq x \leq c \\ \alpha & \text{, if } e \leq x \leq f \end{cases}.$$

Obviously $T$ is a cyclical map. We want to find out in which conditions this is a weak cyclic Kannan operator.

Suppose $c < e$. Then $D(A, B) = e - c$. Let $x \in A$ and $y \in B$. In this particular situation condition (5) becomes:

$$\beta - \alpha \leq a(y - x + \beta - \alpha) + (1 - 2a)(e - c). \tag{6}$$

As $b \leq x \leq c$ and $e \leq y \leq f$ it follows that $(y-x+\beta-\alpha) \in [e-c+\beta-\alpha, f-b+\beta-\alpha]$. Since the minimum value of the right hand in (6) is $a(e-c+\beta-\alpha)+(1-2a)(e-c)$, then for (6) to hold it is necessary that

$$\beta - \alpha \leq a(e - c + \beta - \alpha) + (1 - 2a)(e - c),$$

which implies $\beta - \alpha \leq e - c$, that is true only for $\beta = e$ and $\alpha = c$.

If $c = e$ then $D(A, B) = 0$ and for all $x \in A, y \in B$, (5) becomes:

$$\beta - \alpha \leq a(y - x + \beta - \alpha). \tag{7}$$

Since the minimum value of the right side in (7) is $a(\beta - \alpha)$, so for (7) to hold it is necessary that $\beta - \alpha \leq a(\beta - \alpha)$, which is impossible since $a \in \left(0, \frac{1}{2}\right)$.

Analyzing these situations we conclude that in order for (5) to be full filed, $T$ must be defined by:

$$Tx = \begin{cases} e & \text{, if } b \leq x \leq c \\ c & \text{, if } e \leq x \leq f \end{cases},$$

where $0 \leq b \leq c < e \leq f$, are real numbers. It is easy to see that $T$ is also a cyclic contraction.

**Example 3.** Let $A = B = [0,1]$ be two subsets of $X = \mathbb{R}$ with the usual norm, and a map defined by $Tx = \dfrac{x}{3}$, for all $x \in [0,1]$. Then $D(A,B) = 0$. $T$ is cyclic contraction but not a weak cyclic Kannan contraction (see [8]).

**Example 4.** Let $A = B = \mathbb{R}$ be two subsets of $X = \mathbb{R}$ with the usual norm, and a map $T : \mathbb{R} \to \mathbb{R}$ defined by

$$Tx = \begin{cases} 0 & , \text{ if } x \in (-\infty, 2] \\ -\frac{1}{2} & , \text{ if } x \in (2, \infty) \end{cases},$$

Then $D(A,B) = 0$. $T$ is not a cyclic contraction but $T$ is a weak cyclic Kannan contraction (see [1]).

In order to obtain the existence and uniqueness of a best proximity point we will use two important convergence lemmas taken from [3].

**Lemma 1.** *([3]) Let $A$ be a nonempty closed and convex subset and $B$ be a nonempty closed subset of a uniformly convex Banach space. Let $\{x_n\}$ and $\{z_n\}$ be sequences in $A$ and $\{y_n\}$ be a sequence in $B$ satisfying:*

*(i)* $\|z_n - y_n\| \to D(A,B)$*;*

*(ii) For every $\epsilon > 0$ there exists $N_0$ such that for all $m > n \geq N_0$, $\|x_m - y_n\| \leq D(A,B) + \epsilon$.*

*Then, for every $\epsilon > 0$ there exists $N_1$ such that for all $m > n \geq N_1$,*

$$\|x_m - z_n\| \leq \epsilon.$$

**Lemma 2.** *([3])Let $A$ be a nonempty closed and convex subset and $B$ be nonempty closed subset of a uniformly convex Banach space. Let $\{x_n\}$ and $\{z_n\}$ be sequences in $A$ and $\{y_n\}$ be a sequence in $B$ satisfying:*

*(i)* $\|x_n - y_n\| \to D(A,B)$*;*

*(ii)* $\|z_n - y_n\| \to D(A,B)$.

*Then $\|x_n - z_n\|$ converges to zero.*

Taking into account these results and the forthcoming in this paper, we shall need the following lemma.

**Lemma 3.** *Let $X$ be a uniformly convex Banach space, $A$ and $B$ be two nonempty, closed, convex subsets of $X$. Let $T : A \cup B \to A \cup B$ be a cyclic operator satisfying*

$$\|Tx - T^2x\| \leq \alpha\|x - Tx\| + (1-\alpha)D(A,B), \text{ for all } x \in A \cup B, \tag{8}$$

*where $\alpha \in [0,1)$. Then*

*(i)* $\|T^n x - T^{n+1}x\| \leq \alpha^n\|x - Tx\| + (1-\alpha^n)D(A,B)$*, for all $x \in A \cup B$ and $n \geq 0$;*

*(ii)* $\|T^n x - T^{n+1}x\| \to D(A,B)$, *as* $n \to \infty$, *for all* $x \in A \cup B$;

*(iii)* $\|T^{2n}x - T^{2n\pm 2}x\| \to 0$, *as* $n \to \infty$, *for all* $x \in A \cup B$.

*(iv)* $z$ *is a best proximity point if and only if* $z$ *is a fixed point of* $T^2$.

*Proof.* The first statement follows immediately by induction, using (8). Letting $n \to \infty$ in (i) yields the second statement (ii).

To prove (iii) we will use Lemma 2 and (ii). Let $x \in A \cup B$. Then by (ii) it follows that $\|T^{2n}x - T^{2n-1}x\| \to D(A,B)$ and $\|T^{2n-2}x - T^{2n-1}x\| \to D(A,B)$. From Lemma 2 it results

$$\|T^{2n}x - T^{2n-2}x\| \to D(A,B)$$

for any $x \in A \cup B$. Similarly, $\|T^{2n}x - T^{2n+1}x\| \to D(A,B)$ and $\|T^{2n+2}x - T^{2n+1}x\| \to D(A,B)$, and then by Lemma 2 we obtain

$$\|T^{2n}x - T^{2n+2}x\| \to D(A,B).$$

We next prove (iv). First we assume that $z$ is a best proximity point, i.e. $\|z - Tz\| = D(A,B)$. Then, from (i) we have $\|T^2 z - Tz\| = D(A,B)$. By Lemma 2, we obtain $T^2 x = z$. Now, assume that $z$ is a fixed point of $T^2$ and $z$ is not a best proximity point, i.e., $D(A,B) < \|z - Tz\|$. Then, using (8) we have:

$$\begin{aligned} \|z - Tz\| &= \|T^2 z - Tz\| \leq \alpha\|z - Tz\| + (1-\alpha)D(A,B) \\ &< \alpha\|z - Tz\| + (1-\alpha)\|z - Tz\| = \|z - Tz\|, \end{aligned}$$

a contradiction. This completes the proof. □

**Remark 1.** The uniform convexity of the space $X$ was needed to prove (iii) and (iv). Therefore (i) and (ii) can be used even if we have a metric space setting.

# 3 Main results

In this section we prove our main result. We begin this section by pointing out the importance of Lemma 3.

**Remark 2.** If we take $y := Tx$ in (5) then we obtain (8), where $\alpha = \dfrac{a}{1-a}$. Hence we can apply Lemma 3. Since $a \in \left[0, \dfrac{1}{2}\right)$, it is easy to see that $\alpha \in [0,1)$.

The following result gives a necessary condition for the existence of a best proximity point, for a weak cyclic Kannan mapping.

**Theorem 4.** *Let* $(X,d)$ *be a metric space, let* $A$ *and* $B$ *be nonempty subsets of* $X$ *and let* $T : A \cup B \to A \cup B$ *be a weak cyclic Kannan contraction. Let* $x \in A$ *such that a subsequence* $\{T^{2n_i}x\}$ *of* $\{T^{2n}x\}$ *converges to* $z \in A$. *Then* $z$ *is the unique best proximity point of* $T$.

*Proof.* Let $x \in A$ such that $\lim_{i\to\infty} T^{2n_i}x = z$. Using (5), Remark 2 and the statement (ii) from Lemma 3, we have

$$\begin{aligned} d(z,Tz) &= \lim_{i\to\infty} d(T^{2n_i}x, Tz) \leq \\ &\leq \lim_{i\to\infty} a[d(T^{2n_i-1}x, T^{2n_i}x) + d(z,Tz)] + (1-2a)D(A,B) \\ &= ad(z,Tz) + (1-a)D(A,B). \end{aligned}$$

Hence $d(z,Tz) \leq D(A,B)$, so $d(z,Tz) = D(A,B)$. Arguing by contradiction, we will prove that $z$ is the unique best proximity point of $T$. Let $z'$ be another best proximity point, i.e. $d(z',Tz') = D(A,B)$. Then by Lemma 3 we note that $z' = T^2z'$. We assume that $d(z',Tz) > D(A,B)$. Then, using (5) and the above considerations, it results

$$\begin{aligned} d(z',Tz) &= d(T^2z', Tz) \\ &\leq a[d(z',Tz') + d(z,Tz)] + (1-2a)D(A,B) \\ &= 2aD(A,B) + (1-2a)D(A,B) = D(A,B), \end{aligned}$$

a contradiction. Hence $d(z',Tz) = D(A,B)$. Now, from $d(z,Tz) = D(A,B)$ and Lemma 2 it follows that $z = z'$. This completes the proof. □

Now we are able to prove our main result.

**Theorem 5.** *Let $A$ and $B$ be nonempty closed convex subsets of a uniformly convex Banach space. Suppose $T : A \cup B \to A \cup B$ is a weak cyclic Kannan contraction map. Then*

*(i) $T$ has a unique best proximity point $z$ in $A$.*

*(ii) The sequence $\{T^{2n}x\}$ converges to $z$ for any starting point $x \in A$.*

*(iii) $z$ is the unique fixed point of $T^2$.*

*(iv) $Tz$ is a best proximity point of $T$ in $B$.*

*Proof.* Suppose $D(A,B) = 0$, then $A \cap B \neq \emptyset$ and the theorem follows from Kannan fixed point, Theorem 2, as $T$ is a Kannan operator on $A \cap B$.

Therefore, we assume $D(A,B) \neq 0$. Let $x \in A$. Since (5) implies (8), (see Remark 2), by (ii) from Lemma 3 we have that

$$\|T^{2n}x - T^{2n+1}x\| \to D(A,B).$$

If, for given $\epsilon > 0$, there exists an $N_0 \in \mathbb{N}$ such that for $m > n \geq N_0$,

$$\|T^{2m}x - T^{2n+1}x\| \leq D(A,B) + \epsilon,$$

then by Lemma 1 and for given $\epsilon > 0$, there exists an $N_1$, such that, for $m > n \geq N_1$, $\|T^{2m}x - T^{2n}x\| \leq \epsilon$. Then $\{T^{2n}x\}$ is a Cauchy sequence and hence a convergent one. Let $z \in A$ such that $T^{2n}x \to z$, as $n \to \infty$. By Theorem 4, $z$ is the unique best proximity point of $T$ in $A$. By Lemma 3 $z$ is the unique fixed point of $T^2$, since $z$ is unique.

Now, assume the contrary: there exists an $\epsilon_0 > 0$, such that for every $k \in \mathbb{N}$, there exists $m_k > n_k \geq k$ such that

$$\|T^{2m_k}x - T^{2n_k+1}x\| > D(A,B) + \epsilon_0. \tag{9}$$

Let $m_k$ be the smallest integer greater than $n_k$, to satisfy the above inequality, i.e.

$$\|T^{2(m_k-1)}x - T^{2n_k+1}x\| \leq D(A,B) + \epsilon_0.$$

Now, by using the triangle rule, we have

$$D(A,B)+\epsilon_0 < \|T^{2m_k}x - T^{2n_k+1}x\| \leq \|T^{2m_k}x - T^{2m_k-2}x\| + \|T^{2m_k-1}x - T^{2n_k+1}x\|.$$

By Lemma 3 (iii), $\|T^{2m_k}x - T^{2m_k-2}x\| \to 0$, as $k \to \infty$. Therefore,

$$D(A,B) + \epsilon_0 \leq \lim_{k\to\infty} \|T^{2m_k}x - T^{2n_k+1}x\| \leq D(A,B) + \epsilon_0.$$

So, $\lim_{k\to\infty} \|T^{2m_k}x - T^{2n_k+1}x\| = D(A,B) + \epsilon_0$. On the other hand, by triangle inequality, it follows

$$\|T^{2m_k}x - T^{2n_k+1}x\| \leq \|T^{2m_k}x - T^{2m_k+2}x\| + \|T^{2m_k+2}x - T^{2n_k+3}x\| +$$

$$+\|T^{2n_k+3}x - T^{2n_k+1}x\|.$$

Now, letting $k \to \infty$, and using (iv) from Lemma 3, (5), from the above inequality, we obtain

$$D(A,B) + \epsilon_0 \leq \lim_{k\to\infty} \|T^{2m_k+2}x - T^{2n_k+3}x\| \leq$$

$$\leq \lim_{k\to\infty} a\big[\|T^{2m_k+1}x - T^{2m_k+2}x\| + \|T^{2n_k+2}x - T^{2n_k+3}x\|\big] +$$

$$+(1-2a)D(A,B) = D(A,B).$$

This leads to $\epsilon_0 \leq 0$, a contradiction. We still have to prove that $Tz$ is a best proximity point of $T$ in $B$. Since $z$ is the unique best proximity point we have that $\|z - Tz\| = D(A,B)$. Now, by (i) from Lemma 3, it follows:

$$D(A,B) \leq \|Tz - T^2z\| \leq \alpha\|z - Tz\| + (1-\alpha)D(A,B) = D(A,B),$$

where $\alpha = \dfrac{a}{1-a}$. Therefore, $\|Tz - T^2z\| = D(A,B)$, i.e., $Tz$ is a best proximity point of $T$ in $B$. This completes the proof. □

# 4 Weak $p$-cyclic Kannan contractions

Let $p \in \mathbb{N}, p > 1$. Consider a self map $T$ defined on the union of $p$ nonempty subsets $A_1, A_2, \ldots, A_p$ of a metric space $(X, d)$, satisfying the following *generalized cyclical conditions*

$$T(A_i) \subseteq A_{i+1},$$

for all $1 \leq i \leq p$, where $A_{p+1} = A_1$. In this case $T$ is called a *weak $p$-cyclic Kannan contraction* if it also satisfies the contractive condition

$$d(Tx, Ty) \leq a[d(x, Tx) + d(y, Ty)] + (1 - 2a)D(A_i, A_{i+1}),$$

for all $x \in A_i$ and $y \in A_{i+1}$, for each $1 \leq i \leq p$, where $a \in \left(0, \frac{1}{2}\right)$.

In a similar manner one can extend the main result of the previous section, namely Theorem 5, which is formulated for $p = 2$, to the case $p \geq 2$.

**Theorem 6.** *Let $\{A_i\}_{i=1}^p$ be nonempty, closed and convex subsets of a uniformly convex Banach space. Let $T : \bigcup_{i=1}^{p} A_i \to \bigcup_{i=1}^{p} A_i$ be a weak $p$-cyclic Kannan contraction. Then*

*(i) for each $i, 1 \leq i \leq p$, there exists a unique best proximity point $x_i^*$, of $T$ in $A_i$;*

*(ii) the sequence $\{T^{pn}x\}$ converges to $x_i^*$ for any starting point $x \in A_i$;*

*(iii) $x_i^*$ is the unique fixed point of $T^p$;*

*(iv) $T^j x_i^* = x_{i+j}^*$ is a best proximity point of $T$ in $A_{i+j}$, for $j = \overline{0, p-1}$.*

The proof of this result will be presented elsewhere.

**Conclusions:**

1. From our Theorem 5 one obtains Theorem 2, as a particular case, when the intersection of the two sets is nonempty.

2. From our Theorem 6 one obtains Theorem 6 in [11] and respectively Theorem 4.1 in [15], as a particular case, when the intersection of a finite number of sets is nonempty.

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