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ON THE CONVERGENCE OF THREE-STEP ITERATIVE SEQUENCES WITH ERRORS FOR NONSELF ASYMPTOTICALLY QUASI-NONEXPANSIVE TYPE MAPPINGS IN BANACH SPACES

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Abstract

In this paper, it is proved that the three-step iteration sequence converges strongly to a fixed point for nonself asymptotically quasi-nonexpansive type mappings in Banach space. The results obtained in this paper extend and improve the recent ones announced by S.S. Chang and Y.Y. Zhou[2] , S. Quan, S.Chang and J.Long [13] , W. Nilsrakoo, S. Saejung [10] and many others.

1 Introduction and Preliminaries

Throughout this paper, we assume that X be a real Banach space, K is a nonempty closed subset X and F(T) is the set of fixed points of mapping T.

Definition 1.1. (1) A mapping T is called *nonexpansive* if $\|Tx-Ty\| \leq \|x-y\|$ for all $x, y \in K$.

(2) T is called *quasi-nonexpansive* if $F(T) \neq \emptyset$ and $\|Tx-p\| \leq \|x-p\|$ for all $x \in K$ and $p \in F(T)$.

(3) T is called *asymptotically nonexpansive* mapping if there exist a sequence $\{\upsilon_n\} \subset [1, \infty)$ with $\lim\limits_{n\to\infty} \upsilon_n = 1$ such that $\|T^n x - T^n y\| \leq \upsilon_n \|x-y\|$ for all $x, y \in K$ and $n \geq 1$.

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(4) T is called *asymptotically quasi-nonexpansive* mapping if $F(T) \neq \emptyset$ and there exist a sequence $\{\upsilon_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} \upsilon_n = 1$ such that
$\|T^n x - p\| \leq \upsilon_n \|x - p\|$ for all $x \in K, p \in F(T)$ and $n \geq 1$.

(5) T is called *asymptotically nonexpansive type* mapping if
$\limsup_{n\to\infty} \{\sup\{\|T^n x - T^n y\|^2 - \|x - y\|^2\}\} \leq 0$ for all $x, y \in K$ and $n \geq 1$.

(6) T is called *asymptotically quasi-nonexpansive type* mapping if
$\limsup_{n\to\infty} \{\sup\{\|T^n x - p\|^2 - \|x - p\|^2\}\} \leq 0$ for all $x \in K, p \in F(T)$ and $n \geq 1$.

From above definitions, it follows that if $F(T)$ is nonempty, a quasi-nonexpansive, asymptotically nonexpansive, asymptotically quasi-nonexpansive and asymptotically nonexpansive type are all special cases of asymptotically quasi-nonexpansive type. But the converse does not hold.

The iterative approximation problems for nonexpansive mapping, asymptotically nonexpansive mapping and asymptotically quasi-nonexpansive mapping were studied Ghosh and Debnath [4], Goebel and Kirk [3], Liu [6]-[7], Petryshyn and Williamson [12] in the settings of Hilbert spaces and uniformly convex Banach spaces. The strong and weak convergence of the sequence of Mann iterates to a fixed point of quasi-nonexpansive mappings were studied by Petryshyn and Williamson [12]. Subsequently, the convergence of Ishikawa iterates of quasi-nonexpansive mappings in Banach spaces were discussed by Ghosh and Debnath [4]. The above results and obtained some necessary and sufficient conditions for Ishikawa iterative sequences to converge to a fixed point for asymptotically quasi-nonexpansive mappings were extended by Liu [6]-[7]. In [2], the convergence theorems for Ishikawa iterative sequences with mixed errors of asymptotically quasi-nonexpansive type mappings in Banach spaces were studied.

In 2000, Noor [11] introduced a three-step iterative scheme and studied the approximate solutions of variational inclusion in Hilbert spaces. Glowinski and Le Tallec [5] proved that the three-step iterative scheme gives better numerical results than the Mann-type(one-step) and the Ishikawa-type (two-step) approximate iterations. Xu and Noor [16] introduced and studied a three-step iterative for asymptotically nonexpansive mappings and they proved weak and strong convergence theorems for asymptotically nonexpansive mappings in a Banach space. Very recently, Nilsrakoo and Saejung [10] defined a new three-step iterations which is an extension of Noor iterations and gave some weak and strong convergence theorems of the modified Noor iterations for asymptotically nonexpansive mappings in Banach space.

The iterative scheme given in [10] is defined as follows.

$$\left\{\begin{array}{l} z_n = a_n T^n x_n + (1 - a_n) x_n \\ y_n = b_n T^n z_n + c_n T^n x_n (1 - b_n - c_n) x_n \\ x_{n+1} = \alpha_n T^n y_n + \beta_n T^n z_n + \gamma_n T^n x_n + (1 - \alpha_n - \beta_n - \gamma_n) x_n, \forall n \geq 1, \end{array}\right. \tag{1.1}$$

where $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{b_n+c_n\}$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ and $\{\alpha_n+\beta_n+\gamma_n\}$ in $[0,1]$ satisfy certain conditions.

If $\{\gamma_n\}=0$, then (1.1) reduces to the modified Noor iterations defined by Suantai [14] as follows:

$$\begin{cases} z_n = a_nT^nx_n + (1-a_n)x_n \\ y_n = b_nT^nz_n + c_nT^nx_n + (1-b_n-c_n)x_n \\ x_{n+1} = \alpha_nT^ny_n + \beta_nT^nz_n + (1-\alpha_n-\beta_n)x_n, \forall n \ge 1, \end{cases} \tag{1.2}$$

where $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{b_n\}+\{c_n\}$, $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\alpha_n\}+\{\beta_n\}$ in $[0,1]$ satisfy certain conditions.

If $\{c_n\}=\{\beta_n\}=\{\gamma_n\}=0$, then (1.1) reduces to Noor iterations defined by Xu and Noor [16] as follows:

$$\begin{cases} z_n = a_nT^nx_n + (1-a_n)x_n \\ y_n = b_nT^nz_n + (1-b_n)x_n \\ x_{n+1} = \alpha_nT^ny_n + (1-\alpha_n)x_n, \forall n \ge 1, \end{cases} \tag{1.3}$$

If $\{a_n\}=\{c_n\}=\{\beta_n\}=\{\gamma_n\}=0$, then (1.1) reduces to modified Ishikawa iterations[9] as follows:

$$\begin{cases} y_n = b_nT^nz_n + (1-b_n)x_n \\ x_{n+1} = \alpha_nT^ny_n + (1-\alpha_n)x_n, \forall n \ge 1, \end{cases} \tag{1.4}$$

If $\{a_n\}=\{b_n\}=\{c_n\}=\{\beta_n\}=\{\gamma_n\}=0$, then (1.1) reduces to Mann iterative process [8] as follows:

$$x_{n+1} = \alpha_nT^nx_n + (1-\alpha_n)x_n, \quad \forall n \ge 1, \tag{1.5}$$

A closed convex subset K of X is called a *retract* of X if there exists a continuous map $P : X \longrightarrow K$ such that $Px = x$ for all $x \in K$. A map $P : X \longrightarrow K$ is called a retraction if $P^2 = P$. In particular, a subset K is called a *nonexpansive retract* of X if there exists a *nonexpansive retraction* $P : X \longrightarrow K$ such that $Px = x$ for all $x \in K$.

The concept of nonself asymptotically nonexpansive mappings was introduced Chidume et al.[1] as the generalization of asymptotically nonexpansive self-mappings and obtained some strong and weak convergence theorems for such mappings as follows: for $x_1 \in K$,

$$\begin{cases} y_n = P(\beta_nT(PT)^{n-1}x_n + (1-\beta_n)x_n) \\ x_{n+1} = P(\alpha_nT(PT)^{n-1}y_n + (1-\alpha_n)x_n), \forall n \ge 1, \end{cases} \tag{1.6}$$

where $\{\alpha_n\}$ and $\{\beta_n\} \subset [\delta, 1-\delta]$ for some $\delta \in (0,1)$.

Next, we introduce the following concepts for nonself mapping.

Let X be a real Banach space. A subset K of X be nonempty nonexpansive retraction of X and P be nonexpansive retraction from X onto K. Let $T : K \longrightarrow X$ be a nonself asymptotically nonexpansive mappings.

A nonself mapping T is called *asymptotically nonexpansive* if there exist a sequence $\{\upsilon_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} \upsilon_n = 1$ such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq \upsilon_n \|x - y\|$$

for all $x, y \in K$ and $n \geq 1$. T is called *uniformly L-Lipschitzian* if there exists constant $L > 0$ such that

$$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq L\|x - y\|$$

for all $x, y \in K$ and $n \geq 1$. From above definition, it is obvious that nonself asymptotically nonexpansive mappings is uniformly L-Lipschitzian.

Next,

$$\limsup_{n\to\infty} \Big(\sup_{x\in X, p\in F(T)} \{\|T(PT)^{n-1}x - p\| - \|x - p\|\} \Big) \leq 0.$$

Observe that

$$\begin{aligned}
\limsup_{n\to\infty} \Big(\sup_{x\in X, p\in F(T)} & \{ \quad \|T(PT)^{n-1}x - p\| - \|x - p\|\} \Big) \\
& \times \ \limsup_{n\to\infty} \Big(\sup_{x\in X, p\in F(T)} \{\|T(PT)^{n-1}x - p\| + \|x - p\|\} \Big) \\
& = \ \limsup_{n\to\infty} \Big(\sup_{x\in X, p\in F(T)} \{\|T(PT)^{n-1}x - p\|^2 - \|x - p\|^2\} \Big) \\
& \quad \leq 0.
\end{aligned}$$

Therefore we have

$$\limsup_{n\to\infty} \Big(\sup_{x\in X, p\in F(T)} \{\|T(PT)^{n-1}x - p\| - \|x - p\|\} \Big) \leq 0.$$

This implies that for any given $\varepsilon > 0$, there exists a positive integer n_0 such that for $n \geq n_0$ we have

$$\Big(\sup_{x\in X, p\in F(T)} \{\|T(PT)^{n-1}x - p\| - \|x - p\|\} \Big) \leq 0.$$

Now, we give the following nonself version of (1.1):

for $x_1 \in K$,

$$\left\{ \begin{array}{l}
z_n = P(a_n T(PT)^{n-1}x_n + (1 - a_n - \psi_n)x_n + \psi_n u_n) \\
y_n = P(b_n T(PT)^{n-1}z_n + c_n T(PT)^{n-1}x_n + (1 - b_n - c_n - \varphi_n)x_n + \varphi_n v_n) \\
x_{n+1} = P(\alpha_n T(PT)^{n-1}y_n + \beta_n T(PT)^{n-1}z_n + \gamma_n T(PT)^{n-1}x_n \\
\qquad\qquad +(1 - \alpha_n - \beta_n - \gamma_n - \phi_n)x_n + \phi_n w_n),
\end{array} \right. \tag{1.7}$$

$\forall n \geq 1$, where $\{a_n\}$, $\{\psi_n\}$, $\{a_n + \psi_n\}$, $\{b_n\}$, $\{c_n\}$, $\{\varphi_n\}$, $\{b_n + c_n + \varphi_n\}$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\phi_n\}$ and $\{\alpha_n + \beta_n + \gamma_n + \phi_n\}$ in $[0, 1]$ and $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ are three bounded sequences in K with the following restrictions:
$\sum\limits_{n=0}^{\infty} \psi_n < \infty$, $\sum\limits_{n=0}^{\infty} \varphi_n < \infty$, $\sum\limits_{n=0}^{\infty} \phi_n < \infty$.

The aim of this paper is to prove the strong theorem of the three-step iterative scheme given in (1.7) to a fixed point for nonself asymptotically quasi-nonexpansive-type mappings in a Banach space. In general, the results presented in this paper improve and extend some recent W. Nilsrakoo and S. Saejung [10] and many others.

In order to prove our main result the following lemma is needed.

Lemma 1.2. *[15] Let $\{a_n\}$ and $\{b_n\}$ be sequences of nonnegative real sequences satisfying the following conditions: $\forall n \geq 1$, $a_{n+1} \leq a_n + b_n$, where $\sum\limits_{n=0}^{\infty} b_n < \infty$. Then*

(i) $\lim\limits_{n\to\infty} a_n$ exists;

(ii) In particular, $\{a_n\}$ has a subsequence $\{a_{n_k}\}$ converging to 0, then $\lim\limits_{n\to\infty} a_n = 0$.

2 Main result

Theorem 2.1. *Let X be a Banach space and K be a nonempty closed subset of Banach space. Let $T : K \longrightarrow X$ be a nonself asymptotically quasi-nonexpansive type in Banach space with $F(T) \neq \emptyset$. Let the sequence $\{x_n\}$ be defined by (1.7). Then $\{x_n\}$ converges strongly to a fixed point of T in K iff*

$$\liminf_{n\to\infty} d(x_n, F(T)) = 0. \tag{2.1}$$

Proof. The necessity of condition (2.1) is obvious.
Next we prove the sufficiency of condition (2.1). Let the sequence $\{x_n\}$ be defined by (1.7). For $p \in F(T)$, by boundedness of the sequences $\{u_n\}, \{v_n\}, \{w_n\}$, we can put

$$M = max\{\sup_{n\geq 1} \|u_n - p\|, \sup_{n\geq 1} \|v_n - p\|, \sup_{n\geq 1} \|w_n - p\|\}.$$

For any given $\varepsilon > 0$, there exists a positive integer n_0 such that $n \geq n_0$ we have

$$\sup_{x\in K, p\in F(T)} \{\|T(PT)^{n-1}x - p\| - \|x - p\|\} < \varepsilon.$$

Since $\{z_n\}, \{y_n\}, \{x_n\} \subset X$, then we have

$$\|T(PT)^{n-1}z_n - p\| - \|z_n - p\| \leq \varepsilon, \forall p \in F(T), \forall n \geq n_0, \tag{2.2}$$

$$\|T(PT)^{n-1}y_n - p\| - \|y_n - p\| \le \varepsilon, \forall p \in F(T), \forall n \ge n_0, \tag{2.3}$$

$$\|T(PT)^{n-1}x_n - p\| - \|x_n - p\| \le \varepsilon, \forall p \in F(T), \forall n \ge n_0. \tag{2.4}$$

Thus for any $p \in F(T)$, using (1.7) and (2.2)-(2.4)

$$\begin{aligned}
\|x_{n+1} - p\| &= \|\alpha_n T(PT)^{n-1}y_n + \beta_n T(PT)^{n-1}z_n + \gamma_n T(PT)^{n-1}x_n \\
&\quad + (1 - \alpha_n - \beta_n - \gamma_n - \phi_n)x_n + \phi_n w_n - p\| \\
&\le \|\alpha_n(T(PT)^{n-1}y_n - p) + \beta_n(T(PT)^{n-1}z_n - p) \qquad (2.5)\\
&\quad +\gamma_n(T(PT)^{n-1}x_n - p) \\
&\quad + (1 - \alpha_n - \beta_n - \gamma_n - \phi_n)(x_n - p) + \phi_n(w_n - p)\| \\
&\le \alpha_n(\|T(PT)^{n-1}y_n - p\| - \|y_n - p\|) + \alpha_n\|y_n - p\| \\
&\quad + \beta_n(\|T(PT)^{n-1}z_n - p\| - \|z_n - p\|) + \beta_n\|z_n - p\| \\
&\quad + \gamma_n(\|T(PT)^{n-1}x_n - p\| - \|x_n - p\|) + \gamma_n\|x_n - p\| \\
&\quad + (1 - \alpha_n - \beta_n - \gamma_n - \phi_n)\|x_n - p\| + \phi_n\|w_n - p\| \\
&\le \alpha_n\varepsilon + \alpha_n\|y_n - p\| + \beta_n\varepsilon + \beta_n\|z_n - p\| + \gamma_n\varepsilon \\
&\quad + (1 - \alpha_n - \beta_n - \phi_n)\|x_n - p\| + \phi_n M.
\end{aligned} \tag{2.6}$$

Consider the second and fourth term in right-hand side of the (2.5), using (1.7), (2.2) and (2.4), we have

$$\begin{aligned}
&\|y_n - p\| \qquad (2.7)\\
&= \|b_n T(PT)^{n-1}z_n + c_n T(PT)^{n-1}x_n + (1 - b_n - c_n - \varphi_n)x_n + \varphi_n v_n - p\| \\
&\le \|b_n(T(PT)^{n-1}z_n - p) + c_n(T(PT)^{n-1}x_n - p) \\
&\quad + (1 - b_n - c_n - \varphi_n)(x_n - p) + \varphi_n(v_n - p)\| \\
&\le b_n(\|T(PT)^{n-1}z_n - p\| - \|z_n - p\|) + b_n\|z_n - p\| \\
&\quad + c_n(\|T(PT)^{n-1}x_n - p\| - \|x_n - p\|) + c_n\|x_n - p\| \\
&\quad + (1 - b_n - c_n - \varphi_n)\|x_n - p\| + \varphi_n\|v_n - p\| \\
&\le b_n\varepsilon + b_n\|z_n - p\| + c_n\varepsilon + (1 - b_n - \varphi_n)\|x_n - p\| + \varphi_n M.
\end{aligned} \tag{2.8}$$

Consider the second in right-hand side of the (2.7), using (1.7) and (2.4), we have

$$\begin{aligned}
\|z_n - p\| &= \|a_n T(PT)^{n-1}x_n + (1 - a_n - \psi_n)x_n + \psi_n u_n - p\| \\
&\le \|a_n(T(PT)^{n-1}x_n - p) + (1 - a_n - \psi_n)(x_n - p) + \psi_n(u_n - p)\| \\
&\le a_n(\|T(PT)^{n-1}x_n - p\| - \|x_n - p\|) + a_n\|x_n - p\| \\
&\quad + (1 - a_n - \delta_n)\|x_n - p\| + \psi_n\|u_n - p\| \\
&\le a_n\varepsilon + (1 - \psi_n)\|x_n - p\| + \psi_n M.
\end{aligned} \tag{2.9}$$

Substituting (2.9) into (2.7) and simplifying, we have

$$\|y_n - p\| \leq (1 - b_n\psi_n - \varphi_n)\|x_n - p\| + b_n\varepsilon(1 + a_n) + c_n\varepsilon + b_n\psi_n M + \varphi_n M. \tag{2.10}$$

Substituting (2.10) and (2.9) into (2.7) and simplifying, we have

$$\begin{aligned}
\|x_{n+1} - p\| &\leq \alpha_n\varepsilon + \alpha_n\Big[(1 - b_n\psi_n - \varphi_n)\|x_n - p\| \\
&+ \alpha_n b_n\varepsilon(1 + a_n) + c_n\varepsilon + b_n\psi_n M + \varphi_n M\Big] \\
&+ \beta_n\varepsilon + \beta_n\Big[a_n\varepsilon + (1 - \psi_n)\|x_n - p\| + \psi_n M\Big] \\
&+ (1 - \alpha_n - \beta_n - \varphi_n)\|x_n - p\| + \phi_n M \\
&= \Big[\alpha_n(1 - b_n\psi_n - \varphi_n) \qquad\qquad (2.11)\\
&\quad +\beta_n(1 - \psi_n) + (1 - \alpha_n - \beta_n - \varphi_n)\Big]\|x_n - p\| \\
&+ \alpha_n^2 b_n\varepsilon(1 + a_n) + \alpha_n c_n\varepsilon + \alpha_n b_n\psi_n M + \alpha_n\varphi_n M\Big] \\
&+ \beta_n a_n\varepsilon + \beta_n\psi_n M + \gamma_n\varepsilon + \phi_n M \\
&= \Big[1 - \alpha_n b_n\psi_n - \alpha_n\varphi_n - \beta_n\psi_n - \phi_n\Big]\|x_n - p\| \\
&+ \varepsilon\Big[\alpha_n(\alpha_n b_n(1 + a_n) + c_n) + \beta_n a_n + \gamma_n\Big] \\
&+ M\Big[\alpha_n b_n\psi_n + \alpha_n\varphi_n + \beta_n\psi_n + \phi_n\Big] \\
&\leq \|x_n - p\| + 5\varepsilon + (2\psi_n + \varphi_n + \phi_n)M.
\end{aligned} \tag{2.12}$$

Let $S_n = 5\varepsilon+(2\psi_n+\varphi_n+\phi_n)M$. Then $\sum\limits_{n=0}^{\infty} S_n < \infty$ since $\sum\limits_{n=0}^{\infty} \psi_n < \infty$, $\sum\limits_{n=0}^{\infty} \varphi_n < \infty$, $\sum\limits_{n=0}^{\infty} \phi_n < \infty$. Then by (2.11), we have

$$\inf_{p\in F(T)} \|x_{n+1} - p\| \leq \inf_{p\in F(T)} \|x_n - p\| + S_n, \forall n \geq n_0. \tag{2.13}$$

From (2.13) and $\sum\limits_{n=0}^{\infty} S_n < \infty$, one can write

$$d(x_{n+1}, F(T)) \leq d(x_n, F(T)) + S_n. \tag{2.14}$$

By Lemma 1.2 and from (2.14), we can get that $\lim\limits_{n\to\infty} d(x_n, F(T))$ exists. Again by the condition $\liminf\limits_{n\to\infty} d(x_n, F(T)) = 0$, we have

$$\lim_{n\to\infty} d(x_n, F(T)) = 0. \tag{2.15}$$

We now show that $\{x_n\}$ is Cauchy sequence in X. For $n \geq n_0, m \geq 1$ and any $p \in F(T)$, we have

$$\begin{aligned} \|x_{n+m} - p\| &\leq \|x_{n+m-1} - p\| + S_{n+m-1} \\ &\leq \|x_{n+m-2} - p\| + (S_{n+m-1} + S_{n+m-2}) \\ &\leq \ldots \leq \|x_n - p\| + \sum_{k=n}^{n+m-1} S_k. \end{aligned} \tag{2.16}$$

Let $\varepsilon > 0$. From (2.15) and $\sum\limits_{n=0}^{\infty} S_n < \infty$, there exists a positive integer $n_1 \geq n_0$ such that for all $n \geq n_1$, we have

$$d(x_{n_1}, F(T)) < \frac{\varepsilon}{4}, \tag{2.17}$$

and

$$\sum_{n=n_1}^{\infty} S_n < \frac{\varepsilon}{4}. \tag{2.18}$$

By (2.17) and the definition of infimum, there exists $p_1 \in F(T)$ such that

$$\|x_{n_1} - p_1\| \leq \frac{\varepsilon}{4}. \tag{2.19}$$

Combining (2.16), (2.17), (2.18) and (2.19), for any $m \geq 1$ and any $p_1 \in F(T)$, we have

$$\begin{aligned} \|x_{n+m} - x_n\| &\leq \|x_{n+m} - p_1\| + \|x_n - p_1\| \\ &\leq \|x_{n_1} - p_1\| + \sum_{k=n_1}^{n+m-1} S_k + \|x_{n_1} - p_1\| + \sum_{k=n_1}^{n-1} S_k \\ &\leq 2\Big(\|x_{n_1} - p_1\| + \sum_{k=n_1}^{\infty} S_k\Big) \\ &\leq 2\Big(\frac{\varepsilon}{4} + \frac{\varepsilon}{4}\Big) = \varepsilon, \end{aligned} \tag{2.20}$$

which implies that $\{x_n\}$ is Cauchy sequence in X. Since X is complete, there exists a $p \in X$ such that $x_n \to p$ as $n \to \infty$. It is easily to see that $F(T)$ is closed. Then from (2.15)and from $x_n \to p$ as $n \to \infty$, the continuity of $d(x_n, F(T)) \to 0$ implies that $d(p, F(T)) \to 0$. Hence $p \in F(T)$.

This completes the proof of Theorem 2.1. □

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